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Measure of Slope Rotatability for Second Order Response Surface Designs under Intra-class Correlated Structure of Errors Using Pairwise Balanced Designs

Sulochana Beeraka[1*] and Victorbabu B. Re[1]

[1]Department of Statistics, Acharya Nagarjuna University, Guntur-522510, Andhra Pradesh, India.

Authors' contributions

This work was carried out in collaboration between both authors. Author SB designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author VBR managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, measure of slope rotatability for second order response surface designs using pairwise balanced designs under intra-class correlated structure of errors is suggested and illustrated with examples.

Keywords: *Response surface design; slope-rotatability; intra-class correlated structure of errors; pairwise balanced designs; weak slope rotatability region.*

1 Introduction

Response surface methodology is a collection of mathematical and statistical techniques useful for analysing problems where several independent variables influence a dependent variable. The independent variables are often called the input or explanatory variables and the dependent variable is often the response variable. An important step in development of response surface designs was the introduction of rotatable designs by Box and Hunter [1]. Das and Narasimham [2] constructed rotatable designs using balanced incomplete block

**Corresponding author: E-mail: Sulochana.statistics@gmail.com;*

designs (BIBD). The study of rotatable designs mainly emphasized on the estimation of absolute response. Estimation of response at two different points in the factor space will often be of great importance. If differences at two points close together, estimation of local slope (rate of change) of the response is of interest. In the design of experiments for estimating the slope of the response surface, slope rotatability is a desirable property. Hader and Park [3] extended the notion of rotatability to cover the slope for the case of second order models. In view of slope rotatability of response surface methodology, a good estimation of derivatives of the response function is more important than estimation of mean response. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses, rate of disintegration of radioactive material in animal etc. cf. Park [4]. Victorbabu and Narasimham [5,6] studied second order slope rotatable designs (SOSRD) using BIBD and pairwise balanced designs (PBD) respectively. Victorbabu [7,8] suggested SOSRD symmetrical unequal block arrangements (SUBA) with two unequal block sizes a review on SOSRD. To access the degree of slope rotatability Park and Kim [9] introduced a measure for second order response surface designs. Park et.al [10] introduced measure of rotatability for second order response surface designs. Surekha and Victorbabu [11,12,13,14] studied measure of slope rotatability for second order response surface designs using central composite designs (CCD), BIBD, PBD and SUBA with two unequal block sizes respectively.

Many authors have studied rotatable designs and slope rotatable designs assuming errors to be uncorrelated and homoscedastic. However, it is not uncommon to come across practical situations when the errors are correlated, violating the usual assumptions. Panda and Das [15] introduced robust first order rotatable designs. Das [16,17,18] introduced and studied robust second order rotatable designs. Das [19] introduced slope rotatability with correlated errors and gave conditions for the different variance-covariance error structures. Das and Park [20] introduced measure of robust rotatability for second order response surface designs. To access the degree of slope rotatability for correlated errors a new measure for second order response surface designs was introduced by Das and Park [21]. Rajyalakshmi and Victorbabu [22,23] studied SOSRD under intra-class structure of errors using SUBA with two unequal block sizes and BIBD respectively. Rajyalakshmi et al. [24] studied SOSRD under intra-class structure of errors using PBD. Sulochana and Victorbabu [25-29] studied SOSRD under intra-class structure of errors using a pair of BIBD, a pair of SUBA with two unequal block sizes, partially balanced incomplete block type designs and measure of slope rotatability for second order response surface designs using CCD and BIBD under intra-class correlated structure of errors respectively.

In this paper, following the works of Park and Kim [9], Das [19,30], Das and Park [21], Surekha and Victorbabu [13], Rajyalakshmi et al. [24] measure of slope-rotatability for second order response surface designs under intra-class correlated structure of errors using PBD for $6 \le v \le 15$ (v number of factors) is suggested.

2 Conditions for Slope Rotatability for Second Order Response Surface Designs for Uncorrelated Errors

The second order surface model $D = (x_{\mu i})$ is

$$y_\mu = b_0 + \sum_{i=1}^{v} b_i x_i + \sum_{i=1}^{v} b_{ii} x_{\mu i}^2 + \sum \sum_{i<j=1}^{v} b_{ij} x_{\mu i} x_{\mu j} + e_\mu; 1 \le \mu \le N \tag{2.1}$$

where x_{ui} denotes the level of the $i^{th} (i = 1,2,...,v)$ factor in the $\mu^{th} (\mu = 1,2,...,N)$ run of the experiment, $e_\mu{}'s$ are correlated errors. Here b_0, b_i, b_{ii}, b_{ij} are the parameters of the model and y_μ is the observed response at the μ^{th} design point. The design is said to be SOSRD if the variance of the estimate of first order partial derivative $y_\mu(x_1, x_2, ..., x_v)$ with respect to each of independent variable x_i is only a function of the distance $s^2 = \sum_{i=1}^{v} x_{\mu i}^2$ of the point $(x_1, x_2, ..., x_v)$ from the origin (centre) of the design. Such a spherical variance function for estimation of slopes in the second order response surface is achieved if the design points satisfy the following conditions

1. $\sum_{\mu=1}^{N} x_{\mu i_1}^{\alpha_1} x_{\mu i_2}^{\alpha_2} x_{\mu i_3}^{\alpha_3} x_{\mu i_4}^{\alpha_4} = 0$; for any α_i odd and $\sum_{i=1}^{N} \alpha_i \leq 4$

. (i) $\sum_{\mu=1}^{N} x_{\mu i}^2 = \text{constant} = N\gamma_2;\ 1 \leq i \leq v$; and

(ii) $\sum_{\mu=1}^{N} x_{\mu i}^4 = \text{constant} = cN\gamma_4;\ 1 \leq i \leq v$

(iii) $\sum_{\mu=1}^{N} x_{\mu i}^2 x_{\mu j}^2 = \text{constant} = N\gamma_4;\ 1 \leq i, j \leq v,\ i \neq v.$

(3) $\dfrac{\gamma_4}{\gamma_2^2} > \dfrac{v}{c+v-1}$

$$\gamma_4\left[v(5-c)-(c-3)^2\right]+\gamma_2^2\left[v(c-5)+4\right]=0 \tag{2.2}$$

where c, γ_4 and γ_2 are constants.

The variances and covariances of the estimated parameters are

$$V\left(\hat{b}_0\right) = \frac{\gamma_4(c+v-1)\sigma^2}{N[\gamma_4(c+v-1)-v\gamma_2^2]}$$

$$V\left(\hat{b}_i\right) = \frac{\sigma^2}{N\gamma_2}$$

$$V\left(\hat{b}_{ij}\right) = \frac{\sigma^2}{N\gamma_4}$$

$$V\left(\hat{b}_{ii}\right) = \frac{\sigma^2}{(c-1)N\gamma_4}\left[\frac{\gamma_4(c+v-2)-(v-1)\gamma_2^2}{\gamma_4(c+v-1)-v\gamma_2^2}\right]$$

$$Cov\left(\hat{b}_0,\hat{b}_{ii}\right)=\left[\frac{-\gamma_2\sigma^2}{N[\gamma_4(c+v-1)-v\gamma_2^2]}\right]$$

$$Cov\left(\hat{b}_{ii},\hat{b}_{ii}\right)=\frac{\sigma^2}{(c-1)N\gamma_4}\left[\frac{\gamma_4-\gamma_2^2}{\gamma_4(c+v-1)-v\gamma_2^2}\right] \tag{2.3}$$

and other covariances are vanish.

3 Second Order Response Surface Designs with Correlated Structure of Errors (cf. Das [19,30], Das and Park [21])

The second order surface model $D=(x_{\mu i})$ is

$$y_\mu=b_0+\sum_{i=1}^{v}b_ix_i+\sum_{i=1}^{v}b_{ii}x_{\mu i}^2+\sum\sum_{i<j=1}^{v}b_{ij}x_{\mu i}x_{\mu j}+e_\mu;1\le\mu\le N \tag{3.1}$$

where x_{ui} denotes the level of the $i^{th}(i=1,2,...,v)$ factor in the $\mu^{th}(\mu=1,2,...,N)$ run of the experiment, $e_\mu{}'s$ are correlated errors. Here b_0, b_i, b_{ii}, b_{ij} are the parameters of the model and y_μ is the observed response at the μ^{th} design point.

3.1 Conditions for slope-rotatability for second order response surface designs with correlated errors

Following Das [19,30], Das and Park [21], the necessary and sufficient conditions for slope-rotatability for second order model with correlated errors are as follows.

The estimated response at x_i is given by

$$\hat{y}_\mu=\hat{b}0+\sum_{i=1}^{v}\hat{b}_ix_i+\sum_{i=1}^{v}\hat{b}ii\,x_i^2+\sum\sum_{i<j=1}^{v}\hat{b}ij\,x_ix_j \tag{3.2}$$

For the second order model as in (3.2), we have

$$\frac{\partial\hat{y}_\mu}{\partial x_i}=\hat{b}_i+2\hat{b}_{ii}x_i+\sum_{j=1,j\ne i}^{v}\hat{b}ij\,x_j \tag{3.3}$$

$$V\left(\frac{\partial \hat{y}_\mu}{\partial x_i}\right) = V(\hat{b}_i) + 4x_i^2 V(\hat{b}_{ii}) + 4x_i \mathrm{Cov}(\hat{b}_i, \hat{b}_{ii})$$

$$+ \sum_{j=1, i\neq j}^{v} x_j^2 V(\hat{b}_{ij}) + \sum_{j=1, s=1} \sum_{j\neq s\neq i}^{v} x_j x_s \mathrm{Cov}(\hat{b}_{ij}, \hat{b}_{is}) \tag{3.4}$$

$$+2 \sum_{j=1, j\neq i}^{v} \mathrm{Cov}(\hat{b}_i, \hat{b}_{ij}) + 4 \sum_{j=1, j\neq i}^{v} x_i x_j \mathrm{Cov}(\hat{b}_{ii}, \hat{b}_{ij})$$

$$V\left(\frac{\partial \hat{y}_\mu}{\partial x_i}\right) = \vartheta^{i.i} + 4x_i^2 \vartheta^{ii.ii} + 4x_i \vartheta^{i.ii} + \sum_{j=1, i\neq j}^{k} x_j^2 \vartheta^{ij.ij} + \sum_{j=1, s=1} \sum_{j\neq s\neq i}^{k} x_j x_s \vartheta^{ij.is}$$

$$+2 \sum_{j=1, j\neq i}^{k} \vartheta^{i.ij} + 4 \sum_{j=1, j\neq i}^{k} x_i x_j \vartheta^{ii.ij}.$$

The variance of estimated first order derivative with respect to each independent variable x_i as in (2.4) will be a function of $s^2 = \sum_{i=1}^{v} x_i^2$ if and only if,

1) $\vartheta^{i.ii} = 0;\ 1 \leq j \leq v,\ \vartheta^{i.ij} = 0;\ 1 \leq j, j \leq v,\ i \neq j$
2) $\vartheta^{ij.ij'} = 0;\ 1 \leq i \neq j \neq j' \leq v$
   $\vartheta^{ij.ij} = 0;\ 1 \leq i, i \leq v, i \neq v$
3) $\vartheta^{i.i} = \text{constant};\ 1 \leq i \leq v$
4) $\vartheta^{ii.ii} = \text{constant};\ 1 \leq i \leq v$
5) $\vartheta^{ij.ij} = \text{constant};\ 1 \leq i < j \leq v$, and
6) $\vartheta^{ii.ii} = \frac{1}{4}\vartheta^{ij.ij};\ 1 \leq i < j \leq v$ (3.5)

The following are the equivalent conditions of (1) to (5) in (3.5) for slope rotatability in second order correlated errors model (3.2)

1)* (i) $\vartheta_{0.j} = \vartheta_{0.jl} = 0;\ 1 \leq j < l \leq v;$

(ii) $\vartheta_{i.j} = 0;\ 1 \leq i, j \leq v, i \neq j;$

(iii) a) $\vartheta_{ii.j} = 0;\ 1 \leq i, j \leq v;$

b) $\vartheta_{i.jl} = 0;\ 1 \leq i, j < l \leq v;$

c) $\vartheta_{ii.jl} = 0;\ 1 \le i, j < l \le v,\ (j,l) \ne (i,j)$

d) $\vartheta_{ij.lt} = 0;\ 1 \le i,\ 1 < j,\ t \le v,\ (i,j) \ne (l,t)$

2)* (i) $\vartheta_{0.jj} = \text{constant} = a_1$, say; $1 \le \mathrm{i} \le v$

(ii) $\vartheta_{i.i} = \text{constant} = \dfrac{1}{g}$, say; $1 \le \mathrm{i} \le v$

(iii) $\vartheta_{ii.ii} = \text{constant} = \eta\left(\dfrac{2}{f} + e\right)$, say; $1 \le i \le v$

3)* (i) $\vartheta_{ii.jj} = \text{constant} = e$, say; $1 \le i, j \le v,\ i \ne j$

(ii) $\vartheta_{ij.ij} = \text{constant} = \dfrac{1}{f}$, say; $1 \le i < j \le v$ (3.6)

where a_1, g, f, e, η are constants.

The variances and covariances of the estimated parameters of the model (3.2) for the slope-rotatability are as follows:

$$V\left(\hat{b}_0\right) = \vartheta^{0.0} = \frac{\eta\left(\frac{2}{f} + e\right) + (v-1)e}{B};\ 1 \le i \le v;$$

$$V\left(\hat{b}_i\right) = \vartheta^{i.i} = g;\ 1 \le i \le v;$$

$$V\left(\hat{b}_{ij}\right) = \vartheta^{ij.ij} = f;\ 1 \le i < j \le v;$$

$$V\left(\hat{b}_{ii}\right) = \vartheta^{ii.ii} = \frac{\vartheta_{00}\{\eta(\frac{2}{f} + e) + (v-2)a_1^2\}}{B\{\eta(\frac{2}{f} + e) - e\}};\ 1 \le i \le v;$$

$$Cov\left(\hat{b}_0, \hat{b}_{ii}\right) = \vartheta^{00.ii} = \frac{-a_1}{B};\ 1 \le i \le v;$$

$$Cov\left(\hat{b}_{ii}, \hat{b}_{ij}\right) = \vartheta^{ii.ij} = \frac{a_1^2 - e\vartheta_{00}}{B\{\eta(\frac{2}{f} + e) - e\}};\ 1 \le i \ne j \le v; \qquad (3.7)$$

where $B = \left[\vartheta_{00}\{\eta(\frac{2}{f} + e) + (v-1)e\} - va_1^2\right]$ and the other covariances are zero.

An inspection of the variance of $\hat{b}_0$ shows that a necessary and sufficient condition for the existence of a non-singular second order designs $B > 0$.

$$4)^* \quad B = \left[\vartheta_{00}\{\eta(\frac{2}{f}+e)+(v-1)e\} - va_1^2 \right] > 0. \tag{3.8}$$

For the second order slope rotatability with correlated errors, $V(\hat{b}_{ii}) = \frac{1}{4} V(\hat{b}_{ij})$ i.e., $\vartheta^{ii.ii} = \frac{1}{4}\vartheta^{ij.ij}$. (3.9)

On simplification of (2.9) using (2.7), we get,

$$5)^* \quad \eta\left(\frac{2}{f}+e\right)\left[4\vartheta_{00} - f\vartheta_{00}\eta\left(\frac{2}{f}+e\right) - f\vartheta_{00}g(v-1) + fva_1^2 + \vartheta_{00}gf\right] + \vartheta_{00}g\{4(v-2)+(v-1)fg\} - a_1^2\{4(v-1)+vfg\} = 0. \tag{3.10}$$

From (3.4), using slope rotatability conditions as in (3.6) and (3.7), we derive

$$\begin{aligned} V\left(\frac{\partial \hat{y}_\mu}{\partial x_i}\right) &= g + 4x_i^2\left(\frac{f}{4}\right) + \sum_{j=1, i\neq j}^{v} x_j^2 f \\ &= g + f\sum_{i=1}^{v} x_i^2 \\ &= g + fs^2 \end{aligned} \tag{3.11}$$

where $s^2 = \sum_{i=1}^{v} x_i^2$ and g, f are as in (3.7).

cf. Das [19,30], Das and Park [21]

4 Intra-class Correlated Structure of Errors (cf. Das [16,19,30])

Intra-class structure is the simplest variance-covariance structure which arises when errors of any two observations have the same correlation and each has the same variance. It is also known as uniform correlation structure.

Let ρ is the correlation between errors of any two observations, each having the same variance σ^2. Then intra-class variance covariance structure of errors given by the class:

$$W_0 = \left\{ W_{N\times N(\rho)} = D(e) = \sigma^2\left[(1-\rho)I_N + \rho E_{N\times N}\right] : \sigma > 0, -(N\text{-}1)^{-1} < \rho < 1 \right\}.$$

Here I_N denotes an identity matrix of order N and $E_{N\times N}$ is a $N\times N$ matrix of all elements1.

It was observe that,

$$W^{-1}_{N\times N}(\rho)=\sigma^2\left[(\delta_0-\gamma_0)I_N+\gamma_0 E_{N\times N}\right]$$

where $\delta_0=\dfrac{1+(N\text{-}1)\rho}{(1\text{-}\rho)\{1+(N\text{-}1)\rho\}}$, $\gamma_0=\dfrac{\rho}{(1\text{-}\rho)\{1\text{-}(N\text{-}1)\rho\}}$ and $\rho>(N\text{-}1)^{-1}$.

(cf. Das [16,19,30])

4.1 Conditions of slope rotatability for second order response surface designs under intra-class correlated structure of errors (cf. Das [19,30])

From (3.6), the necessary and sufficient conditions for the second order slope rotatability under the intra-class structure after some simplifications turn out to be

I $\quad \sum_{\mu=1}^{N}\prod_{\mu=1}^{v} x_{\mu i}^{\alpha_i}=0$; for any α_i odd and $\sum_{i=1}^{N}\alpha_i\leq 4.$

II (i) $\sum_{\mu=1}^{N} x_{\mu i}^{2}=\text{constant}=\text{N}\gamma_2;\ 1\leq i\leq v$; and,

(ii) $\sum_{\mu=1}^{N} x_{\mu i}^{4}=\text{constant}=\text{cN}\gamma_4;\ 1\leq i\leq v,$

III $\quad \sum_{\mu=1}^{N} x_{\mu i}^{2}x_{\mu j}^{2}=\text{constant}=\text{N}\gamma_2;\ 1\leq i, j\leq v,\ i\neq v,$ (4.1)

The parameters of second order slope rotatable design under intra-class structure are as following

$$a_1=\frac{N\gamma_2}{\sigma^2\{1+(N\text{-}1)\rho\}},$$

$$e=\frac{\{1+(N\text{-}1)\rho\}N\gamma_4\text{-}N\gamma_2^2}{\sigma^2(1\text{-}\rho)\{1\text{-}(N\text{-}1)\rho\}},$$

$$\frac{1}{g}=\frac{N\gamma_2}{\sigma^2(1-\rho)},$$

$$\frac{1}{f}=\frac{N\gamma_4}{\sigma^2(1-\rho)}$$

$$\vartheta_{00}=\frac{N}{\sigma^2\{1+(N-1)\rho\}},$$

$$. \qquad (4.2)$$

$$\eta\left(\frac{2}{f}+e\right)=\frac{\eta\{1+(N-1)\rho\}3N\gamma_4-\rho N^2\gamma_2^2}{\sigma^2(1-\rho)\{1+(N-1)\rho\}},$$

where $c=3\eta,\ \gamma_2,\ \gamma_4$ and η are constants.

Note that if $\rho=0,$ (i.e. when errors are uncorrelated and homoscedastic) the conditions (4.1) and (4.2) reduce to

I*: $\sum_{\mu=1}^{N} x_{\mu i_1}^{\alpha_1} x_{\mu i_2}^{\alpha_2} x_{\mu i_3}^{\alpha_3} x_{\mu i_4}^{\alpha_4}=0$; for any α_i odd and $\sum_{i=1}^{N}\alpha_i\le 4$

II*: (i) $\sum_{\mu=1}^{N} x_{\mu i}^2=\text{constant}=N\gamma_2;\ 1\le i\le v$; and

(ii) $\sum_{\mu=1}^{N} x_{\mu i}^4=\text{constant}=cN\gamma_4;\ 1\le i\le v$

III*: $\sum_{\mu=1}^{N} x_{\mu i}^2 x_{\mu j}^2=\text{constant}=N\gamma_4;\ 1\le i,j\le v,\ i\ne v.$ (4.3)

Note that (I), (II) and (III) as in (4.3) are second order slope rotatable conditions when errors are uncorrelated and homoscedastic.

Using (4.2), the expression

$\vartheta_{00}\left[\{\eta\left(\frac{2}{f}+e\right)+(v-1)e\}-va_1^2\right]$ simplifies to

$$\frac{N}{\sigma^2\{1+(N-1)\rho\}}\left[\{c+(v-1)\}N\gamma_4\{1+(N-1)\rho\}-\{\eta+(v-1)\}\rho N^2\gamma_2^2-vN\gamma_2^2\right].$$

The non-singularity condition (3.8) the intra-class structure leads to

$$\left[\{c+(v-1)\}N\gamma_4\{1+(N-1)\rho\}-\{\eta+(v-1)\}\rho N^2\gamma_2^2-vN\gamma_2^2\right]>0 \quad (4.4)$$

where $c=3\eta$.

On simplification of equation (3.10) by using (4.2), we get,

$$\frac{\eta\{1+(N-1)\rho\}3N\gamma_4-\rho N^2\gamma_2^2}{(1-\rho)}\left[\begin{array}{l} 4N-\dfrac{\eta\{1+(N-1)\rho\}3N\gamma_4-\rho N^2\gamma_2^2}{\gamma_4\{1+(N-1)\rho\}}+v\dfrac{N\gamma_2^2(1-\rho)}{\gamma_4\{1+(N-1)\rho)} \\ -(v-2)\dfrac{\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2}{\gamma_4\{1+(N-1)\rho\}} \end{array}\right]$$
$$+\frac{N[\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2]}{(1-\rho)}\left[4(v-2)+(v-1)\frac{\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2}{N\gamma_4\{1+(N-1)\rho\}}\right] \quad (4.5)$$
$$-N^2\gamma_2^2\left[\frac{4(v-1)+\{1+(N-1)\rho\}N\gamma_4-\rho N^2\gamma_2^2}{N\gamma_4\{1+(N-1)\rho}\right]=0.$$

(cf. Das [19])

For $\rho=0,$ (i.e. when errors are uncorrelated and homoscedastic) (4.5) becomes

$$\gamma_4\left[v(5-c)-(c-3)^2\right]+\gamma_2^2\left[v(c-5)+4\right]=0 \quad (4.6)$$

above equation (4.6) is equal to slope rotatability for second order response surface designs with errors are uncorrelated and homoscedastic(cf. Victorbabu and Narasimham [5])

4.2 Slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD (cf. Rajyalakshmi et al. (2020))

Following the works of Hader and Park [3], Victorbabu and Narasimham [5], Das [19,30], Rajyalakshmi et al. [24], the method of slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD is given below. Let $\rho\left(-\frac{1}{N-1}<\rho<1\right)$ be correlation between errors of any two observations, each having the same variance σ^2.

Pairwise balanced designs: The arrangement of v treatments in b blocks will be called a PBD of index λ and type $(v,\ k_1,\ k_2,\ ...,k_m)$ if each block contains $k_1,\ k_2,\ ...,k_m$ treatments $(k_i\le v,\ k_i\ne k_j)$ and each pair of distinct treatments occurs in exactly λ blocks of size $k_i(i=1,2,...,m)$ then $b=\sum_{i=1}^{m} b_i$ and

$\lambda v(v-1)=b=\sum_{i=1}^{m} b_i k_i(k_i-1).$

Let $(v, b, r, k_1, k_2, ..., k_m, \lambda)$, be an equi-replicated PBD, $k = \max(k_1, k_2, ..., k_m)$, Let $2^{t(k)}$ denote a fractional replicate of 2^k in ±1 levels, in which no interaction with less than five factors is confounded. $[1-(v, b, r, k_1, k_2, ..., k_m, \lambda)]$ denote the design points generated from the transpose of incidence matrix of PBD. $[1-(v, b, r, k, \lambda)]2^{t(k)}$ are the $b\ 2^{t(k)}$ design points generated from PBD by 'multiplication' (Raghavarao, 1971). $(a,0,0,...,0)2^1$ denote the design points generated from $(a,0,0,...,0)$ point set, and $\bigcup$ denotes combination of the design points generated from different sets of points. n_0 denote the number of central points. The total number of factorial combinations in the design can be written as $N = bF + 2v + n_0$. Here $F = 2^{t(k)}$.

Result (4.1): For the design points, $[1-(v,b,r,k_1,k_2,...,k_m,\lambda)]F\ U\ (a,0,0,....0)2^1\ U\ (n_0)$ will give a v-dimensional SOSRD under intra-class correlated structure of errors using PBD in $N = bF + 2v + n_0$ design points, where a^2 is positive real root of the fourth degree polynomial equation,

$$\begin{aligned}
&\left[\left(8v - 4N\right)\left(1+(N-1)\rho\right)\right]\left(1+(N-1)\rho\right)a^8 + \left[8vrF\left(1+(N-1)\rho\right)\right]\left(1+(N-1)\rho\right)a^6 + \\
&\left[\left(2vr^2F^2 + \left\{\left(\left(12 - 2v\right)\lambda - 4r\right)N + \left(16\lambda - 20v\lambda + 4vr\right)\right\}F\right)\left(1+(N-1)\rho\right)\right]\left(1+(N-1)\rho\right)a^4 + \\
&\left[\left(4vr + \left(16 - 20v\right)r\lambda\right)\left(1+(N-1)\rho\right)\right]F^2\left(1+(N-1)\rho\right)a^2 + \\
&\left[\left(\left(5v - 9\right)\lambda^2 + \left(6 - v\right)r\lambda - r^2\right)\left(1+(N-1)\rho\right)\right]NF^2 + \\
&\left[\left(vr + 4\lambda - 5v\lambda\right)\left(1+(N-1)\rho\right)\right]\left(1+(N-1)\rho\right)r^2F^3 = 0
\end{aligned}$$

Note: Values of SOSRD under intra-class correlated structure of errors using PBD can be obtained by solving the above equation.

5 Measure of Second Order Slope Rotatability for Correlated Structure of Errors (cf. Das and Park [21])

Following Das and Park [21], equations (3.5), (3.6) and (3.7) give necessary and sufficient conditions for a measure for any general second order response surface designs with correlated errors. Further we have

ϑ^{ii} eual for all i,

$\vartheta^{ii.ii}$ eual for all i,

$\vartheta^{ij.ij}$ eual for all i, j, where $i \neq j$ $\vartheta^{i.ii} = \vartheta^{i.ij} = \vartheta^{ij.ij} = \vartheta^{ij.il} = 0$ for all $i \neq j \neq l$, and for all ρ (5.1)

Das and Park (2009) proposed that, if the conditions in (3.5) together (3.6), (3.7) and (5.1) are met, $M_V(D)$ is the proposed measure of slope rotatability for second order response surface designs for any general correlated error structure.

$$M_v(D) = \frac{1}{1+Q_v(D)}$$

$$\text{where } Q_v(D) = \frac{1}{2(v-1)\sigma^4}\left\{(v+2)(v+4)\sum_{i=1}^{v}\left[(\vartheta^{i.i}-\bar{\vartheta})+\frac{a_i-\bar{a}}{v+2}\right]^2\right.$$

$$+\frac{4}{v(v+2)}\sum_{i=1}^{v}(a_i-\bar{a})^2+2\sum_{i=1}^{v}\left[\left(4\vartheta^{ii.ii}-\frac{a_i}{v}\right)^2+\sum_{i=1;\,j\neq i}^{v}\left(\vartheta^{ij.ij}-\frac{a_i}{v}\right)^2\right]$$

$$+4(v+4)\left(4(\vartheta^{i.ii})^2+\sum_{j=1;j\neq i}^{v}(\vartheta^{i.ij})^2\right)$$

$$\left.+4\sum_{i=1}^{v}\left(4\sum_{j=1;j\neq i}^{v}(\vartheta^{ij.ij})^2\right)+\sum_{j<l;j,l\neq i}\sum^{v}(\vartheta^{ij.il})^2\right\} \tag{5.2}$$

here $\bar{\vartheta}=\frac{1}{v}\sum_{i=1}^{v}\vartheta^{i.i}$, $a_i = 4\vartheta^{ii}+\sum_{j=1;j\neq i}^{v}(\vartheta^{ij.ij})^2$ $(1\le i\le v)$ and $\bar{a}=\frac{1}{v}\sum_{i=1}^{v}a_i$.

It can be easily shown that $Q_v(D)$ in equation (4.2) becomes zero for all values ρ, if and only if the conditions in equations (4.1) hold.

Further, it is simplified to $Q_v(D) = \frac{1}{\sigma^4}\left[4V(b_{ii})-V(b_{ij})\right]^2$. (5.3)

Note that $0 \le M_v(D) \le 1$, and it can be easily shown that $M_v(D)$ is one if and only if the design is slope rotatable with any correlated error structure for all values of ρ, and $M_v(D)$ approaches to zero as the design 'D' deviates from the slope-rotatability under specified correlated error structure.

6 Measure of Slope Rotatability for Second Order Response Surface Designs under Intra-class Correlated Structure of Errors Using Pairwise Balanced Designs

In this paper, the degree of slope rotatability for second order response surface designs under intra-class correlated structure of errors $\left(\rho\left(0\le\rho\le0.9\right)\right)$ using pairwise balanced designs for $6\le v\le 15$ (v number of factors) is suggested.

Following Park and Kim [9], Das and Park [21], Surekha and Victorbabu [13], the proposed measure of slope-rotatability for second order response surface designs under intra-class correlated structure of errors using PBD is given below.

Let $(v, b, r, k_1, k_2, ..., k_m, \lambda)$ denote a PBD. For the design points, $[1-(v, b, r, k_1, k_2, ..., k_m, \lambda)]F\ U\ (a,0,0,....0)2^1\ U\ (n_0)$ will give slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD in $N = bF + 2v + n_0$ design points. For the design points generated from PBD, equations in (3.1) are true. Further, from equations in (3.1), we have,

$$\text{(I)} \quad \sum_{\mu=1}^{N} x_{\mu i}^2 = rF + 2a^2 = N\gamma_2$$

$$\text{(II)} \quad \sum_{\mu=1}^{N} x_{\mu i}^4 = rF + 2a^4 = cN\gamma_4$$

$$\text{(III)} \quad \sum_{\mu=1}^{N} x_{\mu i}^2 x_{\mu j}^2 = \lambda F = N\gamma_4 \tag{6.1}$$

Measure of slope rotatability of second order response surface designs under intra-class correlated structure of errors using PBD can be obtained by

$$M_v(D) = \frac{1}{1+Q_v(D)}$$

$$Q_v(D) = \frac{1}{\sigma^4}\left[4V(b_{ii}) - V(b_{ij})\right]^2$$

$$= \frac{1}{\sigma^4}\left[4\vartheta^{ii.ii} - \vartheta^{ij.ij}\right]^2$$

$$= \frac{1}{\sigma^4}\left[4G - \frac{(1-\rho)\sigma^2}{\lambda F}\right]^2 \tag{6.2}$$

$$where\ G = V(b_{ii}) = \vartheta^{ii.ii}$$

$$= \frac{(1-\rho)\sigma^2}{\left(F(r-\lambda)+2a^4\right)}\left[\frac{N\left((r-\lambda)F+2a^4\right)+(v-1)\left(N\lambda F - r^2\lambda^2 - 4rFa^2 - 4a^4\right)}{N\left((r-\lambda)F+2a^4\right)+(v)\left(N\lambda F - r^2F^2 - 4rFa^2 - 4a^4\right)}\right]$$

By substituting (3.2) and (5.1) in $V(b_{ii})$ of (3.7) we get above G value.

If $M_v(D)$ is one if and only if the design 'D' is slope rotatable under intra-class correlated structure of errors using PBD for all values of ρ, and $M_v(D)$ approaches to zero as the design 'D' deviates from the slope-rotatability under intra-class correlated structure of errors using PBD.

Example: We illustrate the method of measure of slope-rotatability for second order response surface designs under intra-class correlated structure of errors with the help of PBD $(v=6,\ b=7,\ r=3,\ k_1=3,\ k_2=2,\ \lambda=1)$.

The design points, $[1\text{-}(6,7,3,3,2,1)]2^3\ U\ (a,0,0,....0)2^1\ U\ (n_0=1)$ will give a slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD in N = 69 design points for 6 factors. From equations (5.1), we have,

(I) $\sum_{\mu=1}^{N} x_{\mu i}^2 = 24+2a^2 = N\gamma_2$

(II) $\sum_{\mu=1}^{N} x_{\mu i}^4 = 24+2a^4 = cN\gamma_4$

(III) $\sum_{\mu=1}^{N} x_{\mu i}^2 x_{\mu j}^2 = 8 = N\gamma_4$ (6.3)

From (I), (II) and (III) of (6.3), we get $\gamma_2 = \frac{24+2\alpha^2}{69}$, $\gamma_4 = \frac{8}{69}$ and $c=\frac{24+2\alpha^4}{8}$. Substituting γ_2, γ_4 and c in (4.5) and on simplification, we get the following biquadratic equation in a^2.

$$\begin{aligned}&\left[48(1+68\rho)-276(1+68\rho)\right](1+68\rho)a^8+1152(1+68\rho)^2a^6\\&+\left[6912(1+68\rho)-6880(1+68\rho)\right](1+68\rho)a^4-6144(1+68\rho)^2a^2\\&+\left[52992(1+68\rho)-38684(1+68\rho)\right](1+68\rho)=0\end{aligned} \tag{6.4}$$

equation (6.4) has only one positive real root for all values of $a^2 = 4.5314$ This can be alternatively written directly from result (4.1). Solving (6.4), we get $a = 2.1287$ From (6.2) we get $Q_v(D) = 0$, $M_v(D) = 1$ for all values of $\rho(-\frac{1}{N-1} \le \rho \le 0.9)$.

Suppose if we take $a = 1.6$ instead of taking $a = 2.1287$ for the above PBD we get $Q_v(D) = 0.0284$, then $M_v(D) = 0.9723$ (taking $\rho = 0.1$). Here $M_v(D)$ deviates from slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD.

Table 1, gives the values of $M_v(D)$ for second order rotatable designs under intra-class correlated structure of errors using PBD for $\rho(0 \le \rho \le 0.9)$ and $6 \le v \le 15$ (v number of factors).

6.1 Weak slope rotatability region for correlated errors cf. Das and Park [21]

Following Das and Park [21], we also find weak slope rotatability region (WSRR) for second order response surface designs under intra-class correlated structure of errors using PBD.

$M_V(D) \geq d,$

$M_V(D)$ involves the correlation parameter $\rho \in W$ and as such, $M_V(D) \geq d$ for all ρ is too strong to be met. On the other hand, for a given d, we can find range of values of ρ for which $M_V(D) \geq d.$ Das and Park (2009) call this range as the weak slope rotatability region $(\text{WSRR}(\text{R}_{D(d)}(\rho))$ of the design 'D'. Naturally, the desirability of using 'D' will rest on the wide nature of $(\text{WSRR}(\text{R}_{D(d)}(\rho))$ along with its strength d. Generally, we would require 'd' to be very high say, around 0.95 (cf. Das and Park [21]).

Table 2, gives the values of weak slope rotatability region $(\text{WSRR}(\text{R}_{D(d)}(\rho))$ for second order slope rotatable designs under intra-class correlated structure of errors using PBD for $\rho(0 \leq \rho \leq 0.9)$ and $6 \leq v \leq 15$ (v number of factors) respectively.

7 Discussion

In this method, we obtain designs with fewer number of design points. The implications of fewer number of design points leads to effective and reduced cost of experimentation. Here, we may point out this measure of slope rotatability for second order response designs under intra-lass correlated structure of errors using PBD has only 69 design points for v = 6 $(v=6,\ b=6,\ r=3,\ k_1=3,\ \text{k}_2=2, \lambda=1)$ factors, whereas the corresponding measure of slope rotatability for second order response designs under intra-class correlated structure of errors using CCD (v =6) and BIBD $(v=6,\ b=15,\ r=5,\ k=2,\ \lambda=1)$ need 45 and 73 design points respectively.

For v = 10 (v=10,b=11,r=5, k_1=5, k_2=4, λ=2) factors, this method needs 197 design points whereas the corresponding measure of slope rotatability for second order response designs under intra-class correlated structure of errors using CCD (v = 10) and BIBD (v=10,b=45, r=9, k=2, λ=1), need 149 and 201design points respectively.

Table 1. Values of $M_v(D)'s$ for second order slope rotatable designs under intra-class correlated structure of errors using PBD for $\rho(0 \leq \rho \leq 0.9)$ and $6 \leq v \leq 15$ (v number of factors)

$(v=6,\ b=6,\ r=3,\ k_1=3,\ \text{k}_2=2, \lambda=1),\ N=69,\ a^* = 2.1287$										
ρ / a	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	0.9773	0.9815	0.9853	0.9887	0.9917	0.9942	0.9963	0.9979	0.9991	0.9998
1.3	0.9699	0.9756	0.9806	0.9851	0.9889	0.9923	0.9951	0.9972	0.9987	0.9997
1.6	0.9661	0.9724	0.9780	0.9831	0.9875	0.9913	0.9944	0.9968	0.9986	0.9996
1.9	0.9937	0.9949	0.9959	0.9959	0.9977	0.8884	0.9989	0.9994	0.9997	0.9999
2.1287*	1	1	1	1	1	1	1	1	1	1
2.2	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.5	0.9959	0.9967	0.9974	0.9979	0.9985	0.9989	0.9993	0.9996	0.9998	0.9999
2.8	0.9924	0.9938	0.9951	0.9962	0.9972	0.9981	0.9988	0.9993	0.9997	0.9999
3.1	0.9899	0.9918	0.9936	0.9951	0.9964	0.9974	0.9984	0.9991	0.9996	0.9999
3.4	0.9884	0.9906	0.9925	0.9943	0.9958	0.9971	0.9981	0.9989	0.9995	0.9999
3.7	0.9873	0.9897	0.9918	0.9938	0.9954	0.9968	0.9979	0.9988	0.9995	0.9998
4	0.9866	0.9891	0.9914	0.9934	0.9951	0.9966	0.9978	0.9988	0.9995	0.9999

Note 1: *Measure of slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD is calculated by using the formulae (5.2) (Details were provided in Section 6.)*

(v=8, b=15, r=6, k=4, k_2=3, k_3=2, λ=2), N=257, a^*= 2.7066

a \ ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.9981	0.9985	0.9988	0.9991	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999
1.3	0.9979	0.9983	0.9986	0.9989	0.9992	0.9995	0.9997	0.9998	0.9999	0.9999
1.6	0.9977	0.9981	0.9985	0.9989	0.9992	0.9994	0.9996	0.9998	0.9999	0.9999
1.9	0.9982	0.9986	0.9989	0.9991	0.9994	0.9996	0.9997	0.9998	0.9999	0.9999
2.2	0.9993	0.9994	0.9996	0.9997	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
2.5	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.7066*	1	1	1	1	1	1	1	1	1	1
2.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.1	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999

(v=9, b=11, r=5, k_1=5, k_2=3, k_3=2, λ=2), N=195, a^*= 2.8386

a \ ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.9968	0.9974	9980	0.9985	0.9989	0.9992	0.9995	0.9997	0.9998	0.9999
1.3	0.9968	0.9974	0.9979	0.9985	0.9988	0.9992	0.9994	0.9997	0.9998	0.9999
1.6	0.9967	0.9974	0.9979	0.9984	0.9988	0.9992	0.9995	0.9998	0.9999	0.9999
1.9	0.9967	0.9973	0.9979	0.9984	0.9988	0.9992	0.9995	0.9997	0.9999	0.9999
2.2	0.9980	0.9984	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9999	0.9999
2.5	0.9996	0.9997	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1
2.8386*	1	1	1	1	1	1	1	1	1	1
3.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9996	0.9997	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999

(v=10, b=11, r=5, k_1=5, k_2=3, k_3=2, λ=2), N=197, a^*= 2.8928

a \ ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.3	0.9968	0.9974	0.9979	0.9985	0.9988	0.9992	0.9994	0.9997	0.9998	0.9999
1.6	0.9967	0.9974	0.9979	0.9984	0.9988	0.9992	0.9995	0.9998	0.9999	0.9999
1.9	0.9967	0.9973	0.9979	0.9984	0.9988	0.9992	0.9995	0.9997	0.9999	0.9999
2.2	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.5	0.9996	0.9997	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1
2.8386*	1	1	1	1	1	1	1	1	1	1
3.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.4	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9996	0.9997	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999

(v=12, b=16, r=6, k_1=6, k_2=5, k_3=4, k_4=3, λ=2), N=537, a^*= 3.1055

a \ ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.3	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.6	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.9	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.2	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.5	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1
3.1055*	1	1	1	1	1	1	1	1	1	1
3.4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

(v=13, b=16, r=6, k_1=6, k_2=5, k_3=4, k_4=3, λ=2), N=539, a^*= 3.1416

ρ / *a*	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.3	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.6	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.9	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.2	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.5	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1	1	1
3.1416*	1	1	1	1	1	1	1	1	1	1
3.4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

(v=14, b=16, r=6, k_1=6, k_2=5, k_3=4, k_4=3, λ=2), N=541, a^*= 3.1847

ρ / *a*	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.3	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.6	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.9	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.2	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
2.5	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
2.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1
3.1847*	1	1	1	1	1	1	1	1	1	1
3.4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

(v=15, b=16, r=6, k_1=6, k_2=5, k_3=4, k_4=3, λ=2), N=543, a^*= 3.2360

ρ / *a*	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.3	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
1.6	0.9995	0.9996	0.9997	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
1.9	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999
2.2	0.9991	0.9993	0.9994	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
2.5	0.9993	0.9994	0.9995	0.9996	0.9997	0.9998	0.9999	0.9999	0.9999	0.9999
2.8	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.2360*	1	1	1	1	1	1	1	1	1	1
3.4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
4	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Note 2: Here a^* indicates that the values of slope rotatability for second order response surface designs under intra-class correlated structure of errors using PBD

Table 2. Values of WSRRs $R_{D(0.95)}(\rho)$ for second order slope rotatable designs under intra-class correlated structure of errors using PBD for $\rho(0 \le \rho \le 0.9)$ and for $6 \le v \le 15$ (v number of factors)

a	**1**	**1.3**	**1.6**	**1.9**	**2.2**	**2.5**	**2.8**	**3.1**	**3.4**	**3.7**	**4.1**
(6, 7, 3, 3, 2, 1)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
(8, 15, 6, 4, 3, 2, 2)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
(9, 11, 5, 5, 4, 3, 2)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
(10, 11, 5, 5, 4, 3, 2)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
(12, 16, 6, 6, 5, 4, 3, 2)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
(13, 16, 6, 6, 5, 4, 3, 2)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
(14, 16, 6, 6, 5, 4, 3, 2)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9
(15, 16, 6, 6, 5, 4, 3, 2)	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9	0-0.9

Note 3: Measure of weak slope rotatability region for second order response surface designs under intra-class correlated structure of errors is taken from the Table 1 using the formulae

$M_V(D) \geq d$ (where d=0.95) (Details were given in Section 6.1.)

8 Conclusion

In this paper, the measure of slope rotatability for second order response surface designs with intra-class correlated structure of errors using PBD is suggested. The degree of slope rotatability of the given design calculated for different values of $\rho(0 \leq \rho \leq 0.9)$ for $6 \leq v \leq 15$ (v number of factors).

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Competing Interests

Authors have declared that no competing interests exist.

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