Application of Rough Set, GSM and MSM to Analyze Learning Outcome—An Example of Introduction to Education

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Abstract

Introduction to education is one of the basic courses in teacher education professional education, it covers a wide range of subjects. Thus, in order to practice the management teaching goals, the interdisciplinary developed mathematical tools are applied for the study. The participants of this study are students in course of introduction to education, and the research instruments applied are rough set, grey structural modeling (GSM), and matrix based-structural modeling (MSM). The purposes of this paper are: 1) To logically analyze educational datasets to practice the scientific traits in education; 2) To benefit from directed hierarchical analysis to identify and propose action planning; 3) To construct core-oriented educational structure as the criterion-reference for one-lesson-multiple-design and to provide the whole scope and visualized analysis with GSM and MSM.

1. Introduction

"If a person neglects education, he will miss the future". In the 21st century, education carries more the profes-

sionalism and modernity than ever. The course of introduction to education is one of the professional basic education courses, covering a wide range, such as philosophy, psychology, sociology, and shifting territory when accompanied by new elements such as digital migration technology, ecology and cultural anthropology. For students majoring in education of regular training 4 years and those students studying education, they must have been aware of the overarching and comprehensive professional education after one semester, it is clear not simple. Simultaneously, with those responsible for teaching this course, challenges to achieve effective as anticipated clearly not small in the context of learners bring different traits include: motive for registration, belonging to different departments, as college students and graduate students, study attitude are also different [1].

To understand and manage the effectiveness of teaching and learning in course of introduction to education, this study has applied the mathematical tools of research and development towards transdisciplinary research on subjects for students to choose school this subject, aim to: 1) Understand the structural concept of the course focused on the topic of the course and the course, 2) Discriminate the structural conceptual attribute and the development, open this broad concepts, 3) Provide a basis for teaching standards with different textures for the same subject [2]. Research tools were developed based on mathematical principles, such as Rough set, GSM, MSM, and applied in the field of education [3] [4], the results obtained: 1) The value of education practice reflects educational operating with science; 2) Educational thinking is transformed into mathematical logic, structural characterization analysis have to be stratification; 3) Provide exchange channel related educational diagnosis and meta-discourse.

Due to the social requirements of network and postmodernism multi-awareness, effectiveness of education and educational standardization has become the key point interested in the renewal and reform of education. Applying mathematical models to combine the development of science and technology in educational research activities conducive to the development, conversion and modernization as well as receiving the evaluation and supervision from community on education. Therefore, the purpose of the study is to: 1) Recognize the right way of distinctness of the original act in the learning process, build knowledge structures according to the trend of learner as nuclear; 2) This study is scientific and so that possible replicative. The progress of research can be mathematized and visualized from examining the data and calculations, mathematical reasoning to structural analysis; 3) Propose a new model of educational research, that is the educational research project with the implementation process combining qualitative and quantitative analyses.

2. Mathematical Tools

Based on the curriculum of domestic education research methodology, the educational academic and practical researches, educational researches are divided into qualitative and quantitative research paradigms, the former as case study, ethnographic, field study, and narrative analysis, the latter covers the design and construction survey questionnaire, data verification, statistical analysis and prediction. This is to enhance the communication of educational research, the trustworthiness of findings, as well as in response to interdisciplinary trend, the use of mathematically based system of symbolic logic are used as the characterization and analysis tools.

Based on the context of the development of education having coherence and educational research with transferability, using mathematical characterization and analysis tools, in order to: 1) Develop result analysis and interpretation of the data trend; 2) Morphological structural visualization; 3) Propose the design of new researches which have systematic, experimentation, and interpretation. Therefore, the following is introduction of mathematical tools used. This study applied Rough Set Theory of Pawlak [5] and Grey System Theory of Deng [6] as the mathematical models of basic development, intended to mathematize scientific form analysis and characterize the practical problem researches of education.

2.1. Rough Set Theory

The rough set theory was introduced by Pawlak in the early 1980s as a mathematical tool to deal with uncertainty. It does not need to be given quantity description or statistical probability distribution of some characteristics or attributes in advance and does not have to obey any assumptions. Rough set theory only says that the set of objects analyzed imply the knowledge and knowledge is considered a classification ability of objects [7]. "Reduct" and "core" are important concepts in the rough set theory. Reduct is those minimal attribute sets of information system, which keep the same classification capability with original attribute set. The aim of feature subset selection is to find out a minimum set of relevant attributes that describe the dataset as well as all original attributes do. The basic concepts of rough set are introduced in the following section [8] [9].

2.1.1. Information System

Formally, it is an information system, and can be seen as a system:

$$IS = (U, R, V, f) \tag{1}$$

where: i. $U = \{x_1, x_2, \cdots, x_m\}$ is the universe, a non-empty set of finite objects.

ii. $R = \{r_1, r_2, \cdots, r_n\}$ is a non-empty, finite set of attributes.

iii. $V = \{V_1, V_2, \cdots, V_n\}$, V_i is the range of attribute r_i.

iv. f is a set of f_{ri}, information function.

Each attribute $r_i \in R$ defines an information function:

$$f_{r_i} : U \to V_i \tag{2}$$

2.1.2. Indiscernibility Relation

According to rough set theory, for every set of attributes $B \subset A$, an indiscernibility relation $Ind(B)$ is defined: two objects x_i and x_j are indiscernible by the set of attributes B in A, if $b(x_i) = b(x_j)$ for every $b \subset B$. The equivalence class of $Ind(B)$ is called elementary set in B because it represents the smallest discernible groups of objects. For any element x_i of U, the equivalence class of x_i in relation $Ind(B)$ is represented as $[x_i]_{Ind(B)}$.

2.1.3. Upper and Lower Approximations

The rough set approach to data analysis hinges on two basic concepts, namely the lower and the upper approximations of a set, lower approximation refers to the elements that surely belong to the set, and upper approximation refers to the elements that possibly belong to the set.

Let $X \subset U$, the lower approximation of X in B denoted as $\underline{BX}$ ($B \subseteq A$),

$$\underline{BX} = \left\{ x_i \in U \middle| [x_i]_{Ind(B)} \subset X \right\} \tag{3}$$

the upper approximation of the set X is:

$$\overline{BX} = \left\{ x_i \in U \middle| [x_i]_{Ind(B)} \cap X \neq \phi \right\} \tag{4}$$

Some other concepts:

$BNX = \overline{BX} - \underline{BX}$ is called a boundary of X;

$POS_B(X) = \underline{BX}$, called the B-positive region;

$NEG_B(X) = U - \underline{BX}$, called the B-negative region.

2.1.4. Dependence and Significance of Attribute

Let C and D be two indiscernibility relations in set S, C is condition attribute and D is decision attribute.

$$pos_C(D) = \bigcup_{x \in X / Ind(D)} \underline{CX} \tag{5}$$

The dependence $\gamma_C(D)$ of decision attribute D on condition attribute C is defined as:

$$\gamma_C(D) = \frac{pos_C(D)}{|U|} \tag{6}$$

The dependence of attribute decides significance of attribute. The significance of attribute $r_i \in C$ is defined:

$$\sigma_{(C,D)}(r_i) = \frac{\gamma_C(D) - \gamma_{C-\{r_i\}}(D)}{\gamma_C(D)} = 1 - \frac{\gamma_{C-\{r_i\}}(D)}{\gamma_C(D)} \tag{7}$$

2.1.5. Independence of Attributes

If $Ind(R) = Ind(R - r_i)$, then the attribute r_i is called superfluous. Otherwise, the attribute r_i is in dispensable in R. Set B is independent if all its attributes are indispensable.

2.1.6. Core and Reduct of Attributes

If the set of attributes is dependent, it can be interested in finding all possible minimal subsets of attributes, which lead to the same number of elementary sets as the whole set of attributes (called *reducts*) and in finding the set of all indispensable attributes (called *core*). The *reduct* is the essential part of an *IS*, which can discern all objects discernible by the original *IS*. The *core* is the common part of all *reducts*.

$$Core(IS) = \bigcap red(IS) \tag{8}$$

where *red(IS)* is the *reduct* of information system *IS*.

2.2. Grey Structural Modeling (GSM)

GSM was also proposed by Nagai in 2005, which is based on grey system theory and like interpretive structural modeling, its purpose is to analyze structure of system because it can find out the relationship from observed value indirectly such as statistical graphical modeling [10] [11]. GSM was proposed based on proposing grey relational analysis (GRA) to calculate localized grey relational grade, globalized grey relational grade and grey relational ordinal. Then based on grey relational grade to compare with each other, when one of the parties had a larger value, it was identified as more important item, and became criteria of structural system arrangement.

In grey system theory, GRA is an effective mathematical tool to treat the uncertain, multiple, discrete and incomplete information. This study refers the localized grey relational grade which is proposed by Nagai [12].

The original vectors in GRA:

The reference vector x_0 and inspected vectors of original data x_i are established as followed:

$$
\begin{aligned}
x_0 &= \left(x_0(1), x_0(2), \cdots, x_0(n)\right) \\
x_1 &= \left(x_1(1), x_1(2), \cdots, x_1(n)\right) \\
x_2 &= \left(x_2(1), x_2(2), \cdots, x_2(n)\right) \\
&\vdots \\
x_m &= \left(x_m(1), x_m(2), \cdots, x_m(n)\right), \quad \text{where } i = 1, 2, \cdots, m; \ m \in N
\end{aligned}
\tag{9}
$$

The formula of localized grey relational grade is:

$$\gamma_{oi} = \frac{\max\limits_{\forall i} \|x_0 - x_i\|_\rho - \|x_0 - x_i\|_\rho}{\max\limits_{\forall i} \|x_0 - x_i\|_\rho - \min\limits_{\forall i} \|x_0 - x_i\|_\rho} \tag{10}$$

And formula of globalized grey relational grade is:

$$\gamma_{ij} = 1 - \frac{\|x_i - x_j\|_\rho}{\max\limits_{\forall i} \max\limits_{\forall j} \|x_i - x_j\|_\rho} \tag{11}$$

GSM draws digraph by using three parameters: distinguish coefficient ρ which decides the basic composition of digraph, class coefficient θ which gives the hierarchy, and path coefficient ψ which gives an ordered pair of element arrows. In this paper, $\rho = 2$ is used that means Euclidean distance is applied.

2.3. Matrix Based Structure Modeling (MSM)

MSM was proposed by Nagai in 2013 [13] [14], it was intended to generate an overall association between matrices and reconfigure multiple matrices into square design, in order to construct a new relationship between the respective sets of generation, and visualize the entirety of matrix and the logical relationship of local clusters.

Based on the multi-matrix method of interpretative structural modeling, it is simply known as multi-matrix

interpretative structural model.

The combined set of system:

The combined set

$$M = \bigcup M_i \tag{12}$$

where M_i is the concept set matrix, it can be all set of material objects, it can also be the set of all attributes such as set of students, set of questions, set of learning concept, set of goods, set of merchandise category, set of commodity components, set of commodity production conditions, etc.

The structure system theory of MSM is contracted as the following:

Establish the structure system theory of MSM

$$W = MSM(M, T, f) \tag{13}$$

where i. $M = \bigcup M_i$ is a combined matrix of factor set,

ii. $T = \bigcup_{r \in M} T_r$ is structure matrix,

iii. $f : M \times M \to T$ is reachability function,

$$M \times M = \left\{ (m_i, m_j) \big| m_i, m_j \in M \right\}$$

$$f(m_i, m_j) = m_{ij} \in T_r.$$

Because *f* is a reachability function, so *f* satisfies the following:

1) Reflexive law: $f(m_i, m_i) = 1 \Leftrightarrow m_i \to m_i, \forall i$

2) Anti-symmetric law:

$$f(m_i, m_j) = f(m_j, m_i) = 1$$
$$\Leftrightarrow m_i \to m_j, m_j \to m_i \Rightarrow m_i \leftrightarrow m_j \quad, \forall i \neq j$$
$$\Leftrightarrow m_i = m_j$$

3) Transitive law:

$$f(m_i, m_j) = 1, \ f(m_j, m_k) = 1$$
$$\Rightarrow f(m_i, m_k) = 1 \qquad , \forall i \neq j \neq k$$
$$\Leftrightarrow m_i \to m_j, m_j \to m_k$$
$$\Rightarrow m_i \to m_k$$

In practice, the MSM cluster matrix calculates amount and scale size using structural new relation matrix as purpose based on the generating of multiple matrix relation and the creating of organization structure, the first one is derived from the relationship between the various elements of attributes, and second one, it is the cluster related structure of the purpose orientation [15] [16].

3. Analysis of Practical Example

The research objects are 21 third-year students of higher education and 15 syllabus researchers, the former has been the introduction to education of the first year, the latter is chapters in detail of the introduction to education of this semester, aimed to: 1) Understand two learning objects to explain the differences, 2) Form the structure of unit nuclear knowledge, 3) Make a standard reference basis for the teaching of introduction to education in future.

3.1. Research Process

The research process is divided into qualitative and quantitative analysis stages (see **Figure 1**). Firstly, the students select the important concepts related to basic theory of introduction to education; secondly, using the number values of 1 to 9 to fill out the questionnaire and analysis, the greater the value represents the more con-
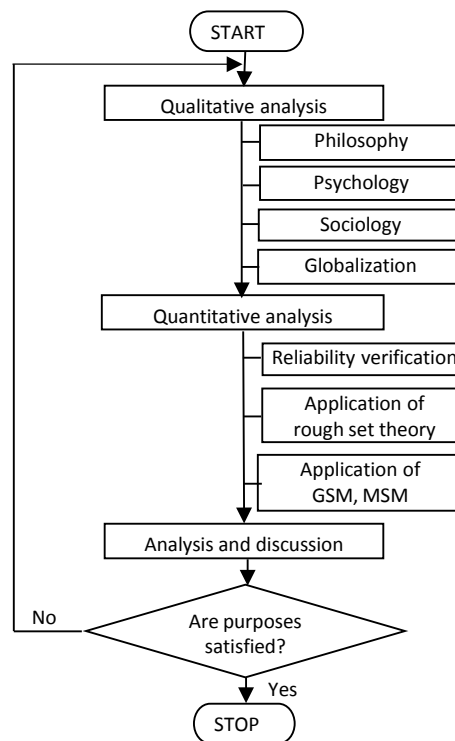
Figure 1. Research process.

cept learning difficulty, on the contrary, the concept learning easier. Thirdly, after testing the reliability of data, using rough set to excavate information; the GSM constitutes isomorphism of "core", analyzes body of knowledge and structure; MSM characterizes its structural relation. Finally, according to the proposal of structural analysis and directed hierarchy diagram, the diagram is explained.

3.2. Q-Matrix

Towards globalization, technology, and Internet of Things (IOT) era, introduction to the basic theory of education from the traditional philosophy, psychology, sociology, new cultural anthropology, ecology, science and technology and culture, etc., are only selecting the concepts on the first four fields as follows (see **Table 1**): 1) Philosophy—epistemology, metaphysics and theory of value; 2) Sociology, cultural anthropology—structural functionalism, system theory, educational political science, educational finance and local/regional culture; 3) Psychology—cognitive science, personality development, positive psychology.

It is worth mentioning: Educational political science and educational finance of sociology aim to: 1) Initiated by the post-modern pluralism, the overall educational practice strategizing becomes necessary, and thus more and more important in educational political science; 2) Facing the globalization of competing interests, profit trends, while input and output performance become the focus of education, on the other hand under the austerity policies being squeezed available funding for education, educational return on investment become truth to be faced; 3) The World is Flat [17], "investing in education, investing in the future," human capital, human development and quality of financial education is closely related.

Next, combining concepts and pedagogy to constitute the Q matrix (**Table 1**), both relations generate principles: 1) Use the binary to represent the relationship between the elements, 2) Distinguish the meanings of concept, expansion of knowledge with its knowledge system and the difficulty of the justification, 3) Teaching model assigns teacher to guide meaningful teaching methods, design the situations of teacher and learners engaged in discovery teaching method, goal setting, problem solving compromise-oriented legislation.

Finally, combining graph theory and interpretive structural modeling (ISM) [18] to build a mapping analysis and visualize the hierarchical structure (see **Figure 2**), conduct the useful classroom, the text with multiple designs, diagnosis and establish the basis for later reflection.
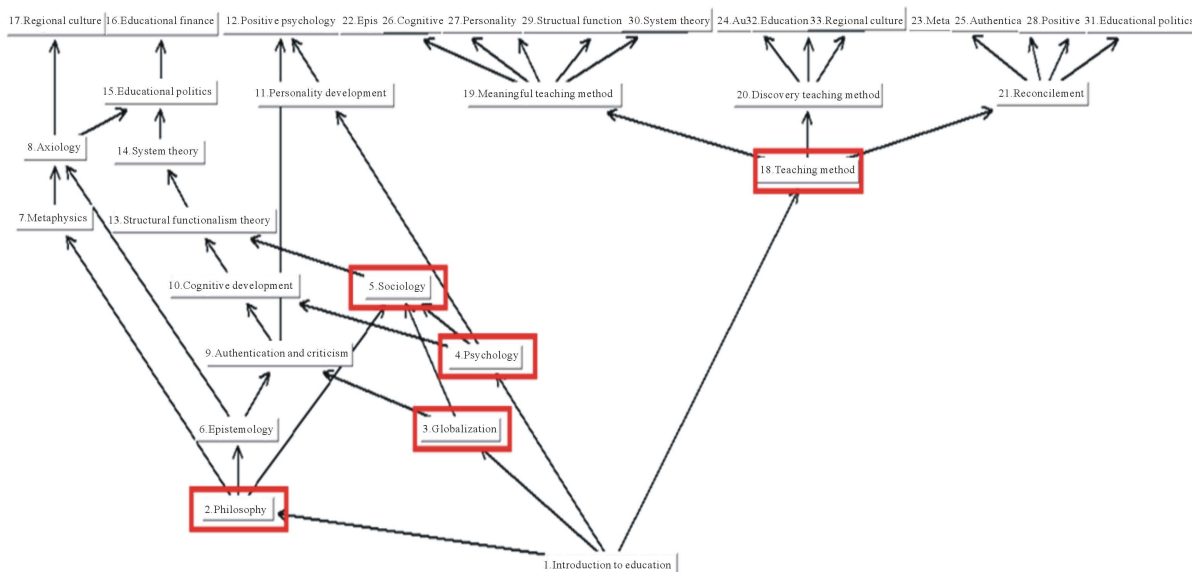
Figure 2. ISM diagram of concepts-teaching method.

Table 1. Q-matrix.

Subject	Concept	Meaning teaching	Discovery teaching	Eclectic teaching
Philosophy	Epistemology	1	0	0
	Metaphysics	0	0	1
	Axiology	0	1	0
Psychology	Cognitive development	1	0	0
	Personality development	1	0	0
	Positive psychology	0	0	1
Sociology	Structural functionalism	1	0	0
	System theory	1	0	0
	Educational politics	0	0	1
	Educational finance	0	1	0
Globalization	Regional culture	0	1	0
	Authenticn and criticism	0	0	1

3.3. Data Processing

Firstly, the data of the four-year undergraduate education including 21 third-year students and 15 syllabus researchers in primary education course, all are tested for reliability, Cronbach's alpha coefficients are shown in **Table 2**, all of them are available for use (clean data). Bryman & Cramer (1997) [19] believed that if Cronbach's alpha coefficient is greater than 0.80 it indicates a set of high reliability data.

Secondly, based on grey relational analysis the degree of relation between the discrete elements which is calculated by formula of Nagai [20], the orders of element are determined (see the **Tables 3-6**). Using ROSE 2 software, from a common intersection of sets of reducts, the "Core" is found out to construct orientation of structural analysis, and identify it as an available teaching reference standard. Moreover, submitting an alternative educational dialogue.

Finally, based on "We Are Student-Centered" (WASC) [21] and implemented performance accomplishments

Table 2. Cronbach's alpha coefficients of sets of data.

Object\table type	S-P table	P-S table
Third year student	0.82	0.86
Researcher	0.82	0.92

Table 3. Localized grey relational grade of educational students/undergraduates.

Student\Concept	A9	A13	A10	A14	A8	A6	A3	A7	A11	A12	A5	A4	Gamma
Max	9	9	9	9	9	9	9	9	9	9	9	7	
S7	9	7	7	8	9	7	9	8	8	9	8	7	**1.00**
S2	8	8	8	9	8	8	8	6	7	7	7	7	**0.93**
S4	9	9	6	5	8	9	5	7	5	6	7	4	**0.69**
S15	9	5	8	3	9	7	8	9	6	7	6	5	**0.69**
S8	8	4	9	4	9	9	8	4	6	7	8	5	**0.66**
S16	7	7	7	7	7	6	5	5	6	6	5	4	**0.63**
S18	7	3	6	5	9	7	8	8	8	5	6	4	**0.62**
S13	8	8	7	8	9	8	6	9	3	2	7	5	**0.61**
S21	8	6	8	7	3	9	3	7	9	7	4	6	**0.58**
S11	9	5	8	3	8	5	5	8	6	7	4	3	**0.53**
S10	9	6	7	5	8	5	5	9	3	2	6	5	**0.48**
S9	6	4	5	6	9	7	3	8	8	9	3	2	**0.48**
S17	7	5	6	5	8	9	4	9	4	3	7	1	**0.45**
S6	8	2	6	4	8	4	7	5	4	4	7	7	**0.43**
S5	8	4	9	4	7	3	7	6	2	3	9	3	**0.38**
S19	9	6	8	3	6	6	6	4	6	2	3	2	**0.36**
S12	7	2	5	3	6	6	5	3	7	7	4	1	**0.30**
S1	4	6	7	7	5	6	5	5	2	3	3	1	**0.28**
S3	6	6	7	7	3	4	2	3	6	4	2	2	**0.24**
S14	6	4	8	2	3	5	2	1	4	1	5	7	**0.11**
S20	5	3	2	4	5	1	5	6	2	1	2	1	**0.00**

Table 4. Localized grey relational grade of concepts in educational student/four-year undergraduates.

Concept\Student	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	Gamma
Max	7	9	7	9	9	8	9	9	9	9	9	7	9	8	9	7	9	9	9	6	9	
A9	4	8	6	9	8	8	9	8	6	9	9	7	8	6	9	7	7	7	9	5	8	**1.00**
A13	6	8	6	9	4	2	7	4	4	6	5	2	8	4	5	7	5	3	6	3	6	**0.92**
A10	7	8	7	6	9	6	7	9	5	7	8	5	7	8	8	7	6	6	8	2	8	**0.91**
A14	7	9	7	5	4	4	8	4	6	5	3	3	8	2	3	7	5	5	3	4	7	**0.79**
A8	5	8	3	8	7	8	9	9	9	8	8	6	9	3	9	7	8	9	6	5	3	**0.71**
A6	6	8	4	9	3	4	7	9	7	5	5	6	8	5	7	6	9	7	6	1	9	**0.50**
A3	5	8	2	5	7	7	9	8	3	5	5	5	6	2	8	5	4	8	6	5	3	**0.49**
A7	5	6	3	7	6	5	8	4	8	9	8	3	9	1	9	5	9	8	4	6	7	**0.45**
A11	2	7	6	5	2	4	8	6	8	3	6	7	3	4	6	6	4	8	6	2	9	**0.43**
A12	3	7	4	6	3	4	9	7	9	2	7	7	2	1	7	6	3	5	2	1	7	**0.41**
A5	3	7	2	7	9	7	8	8	3	6	4	4	7	5	6	5	7	6	3	2	4	**0.27**
A4	1	7	2	4	3	7	7	5	2	5	3	1	5	7	5	4	1	4	2	1	6	**0.00**

Table 5. Localized grey relational grade of master degree program researchers in education.

Student\Concept	A3	A4	A5	A13	A6	A7	A8	A9	A10	A11	A12	A14	Gamma
Max	9	9	9	9	9	9	9	9	9	6	9	7	
S1	9	9	9	9	9	9	9	8	9	6	6	7	**1.00**
S2	9	9	9	6	9	7	6	4	3	6	9	6	**0.58**
S3	9	9	9	7	9	3	7	5	6	4	7	4	**0.58**
S4	8	9	7	9	9	9	6	4	5	6	3	4	**0.53**
S5	9	9	9	9	8	4	5	6	7	3	8	1	**0.52**
S6	8	8	8	9	5	6	7	9	6	4	4	1	**0.51**
S7	9	9	9	6	6	8	9	3	5	4	6	2	**0.50**
S8	9	9	7	7	7	7	7	4	4	2	4	2	**0.41**
S9	8	8	8	4	7	7	4	6	5	5	3	3	**0.40**
S10	9	9	9	5	8	3	1	5	6	4	4	2	**0.25**
S11	7	2	7	8	5	7	6	4	3	2	7	1	**0.23**
S12	8	6	6	6	6	4	6	4	1	2	4	2	**0.18**
S13	3	3	4	3	8	3	4	7	6	3	6	4	**0.16**
S14	9	7	5	4	7	2	1	4	5	3	3	1	**0.05**
S15	6	3	6	6	7	4	7	1	1	6	2	1	**0.00**

Table 6. Localized grey relational grade of concepts in researchers in education/master degree program.

Concept\Student	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	Gamma
Max	9	9	9	9	9	9	9	9	8	9	8	8	8	9	7	
A3	9	9	9	8	9	8	9	9	8	9	7	8	3	9	6	**1.00**
A5	9	9	9	7	9	8	9	7	8	9	7	6	4	5	6	**0.92**
A6	9	9	9	9	8	5	6	7	7	8	5	6	8	7	7	**0.91**
A4	9	9	9	9	9	8	9	9	8	9	2	6	3	7	3	**0.79**
A13	9	6	7	9	9	9	6	7	4	5	8	6	3	4	6	**0.71**
A8	9	6	7	6	5	7	9	7	4	1	6	6	4	1	7	**0.50**
A7	9	7	3	9	4	6	8	7	7	3	7	4	3	2	4	**0.49**
A12	6	9	7	3	8	4	6	4	3	4	7	4	6	3	2	**0.45**
A9	8	4	5	4	6	9	3	4	6	5	4	4	7	4	1	**0.43**
A10	9	3	6	5	7	6	5	4	5	6	3	1	6	5	1	**0.41**
A11	6	6	4	6	3	4	4	2	5	4	2	2	3	3	6	**0.27**
A14	7	6	4	4	1	1	2	2	3	2	1	2	4	1	1	**0.00**

standardization of education is urgently needed. Applying models of educational engineering constructed with mathematical logic in education are adequately proposed approaches.

3.4. Rough Set

This paper has used ROSE2 software of Laboratory of Intelligent Decision Support Systems of the Poznan University of Technology on the URL http://www-idss.cs.put.poznan.pl (2014) [22] [23] for processing data. The data from decision table are inputted, after processing data, the results obtained including: lower approximation, upper approximation, boundary area of attributes, reducts and core of attributes. Each type of results obtained are valuable for research, depending on the purpose of research which types of result and its structural analysis will be interested.

4. Result Analysis

Activities in the classroom, one can see the instructor transmitting knowledge, shows the world we are living is featured in the characterization of the design, on the other hand it shows how the learners think and learn knowledge, judge and understand knowledge in thinking norms, to master the essence of things and phenomena of the world.

It is noteworthy that, the key concepts and knowledge matter to the content, characterization of the content is denoted by the world, the first thing, to acknowledge curriculum, the code is fundamental; the second thing, to identify systematic structure of knowledge manipulating the process of teaching, during shuttling back and forth such as specific vocabulary, interactivity, awareness, misunderstanding or understanding, wrong solution etc., judgment criterion manifests the truth of existence object, reality and adaptability. Based on the complexity and variability of understanding in learning, grasp and diagnostic of learning results are becoming increasingly important.

4.1. Kernel Knowledge Structure

Generally speaking, systematic structure of knowledge transferred in the classroom, one side based on proposition employed with expertise, the other side on academic profession, are implemented with teaching experience and interaction between teachers and students, the transformed principles as following: 1) Between concepts, there are implied relationships, difficulties, the difference in size and logic, 2) The attributes of concept for this hierarchy chain structure from concrete to abstract, from low level rise to high level, and 3) Mindset, cognitive understanding, cognitive learning process and only after understanding the concept below, to learn the advanced concepts and complex difficulties.

Based on the results outputted from ROSE2 software, the third year students and the syllabus researchers, who answered the work sheet, both of them have a simple core (**Table 7**), which will be dealt with, evaluated and diagnosed according to construction of grey structural modeling (see **Figure 2**), the full and simplified forms, provide visual structure analysis.

Table 7. Kernel structure.

Concept\Subject	Researcher	Student
Epistemology	8	3
Axiology	6	3
Cognitive development	6	7
Metaphysics	6	2
Authentic and criticism	6	4
Positive psychology	6	9
Personality development	4	8
Educational finance	4	9
Structural functionalism	4	6
System theory	1	5
Educational politics	2	8
Regional culture	2	6

Figure 3 and Figure 4 and Table 7 show that: 1) For the cognitive understanding difficulties of introduction to education, the structure of the syllabus researchers were denoted more appropriate with judgement, which is the case difficult to understand philosophy section, followed by psychology, sociology belongs to easier learning section; awareness of the third-year students is undifferentiated; 2) For the connotation and denotation of course of introduction to education, the positive psychology belongs to the mental activity of psychological class, has the individual character, philosophy, sociology then has the directions to with real life situations, each focusing on, however, psychology development cannot be divorced philosophy and sociology of norms and context; 3) The system theory, axiology and metaphysics are the basic concept structures, shaping one kind from the whole to the part, from the abstract to the concrete, ordain the thinking pattern of axiology.
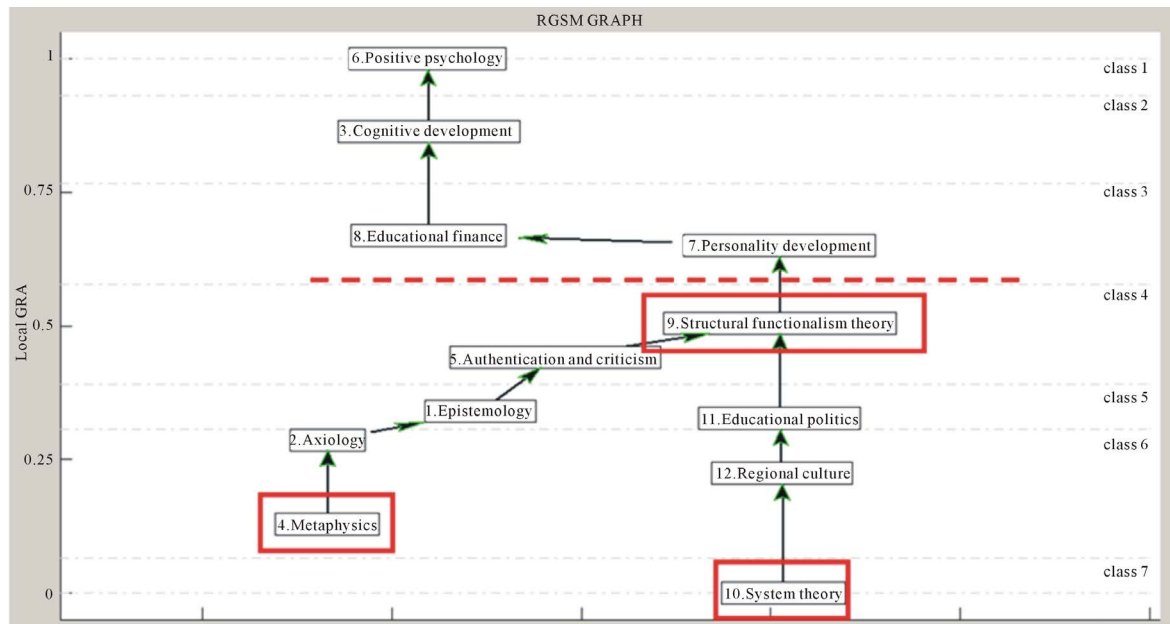
Figure 3. GSM diagram of educational student and researcher (simplified form).
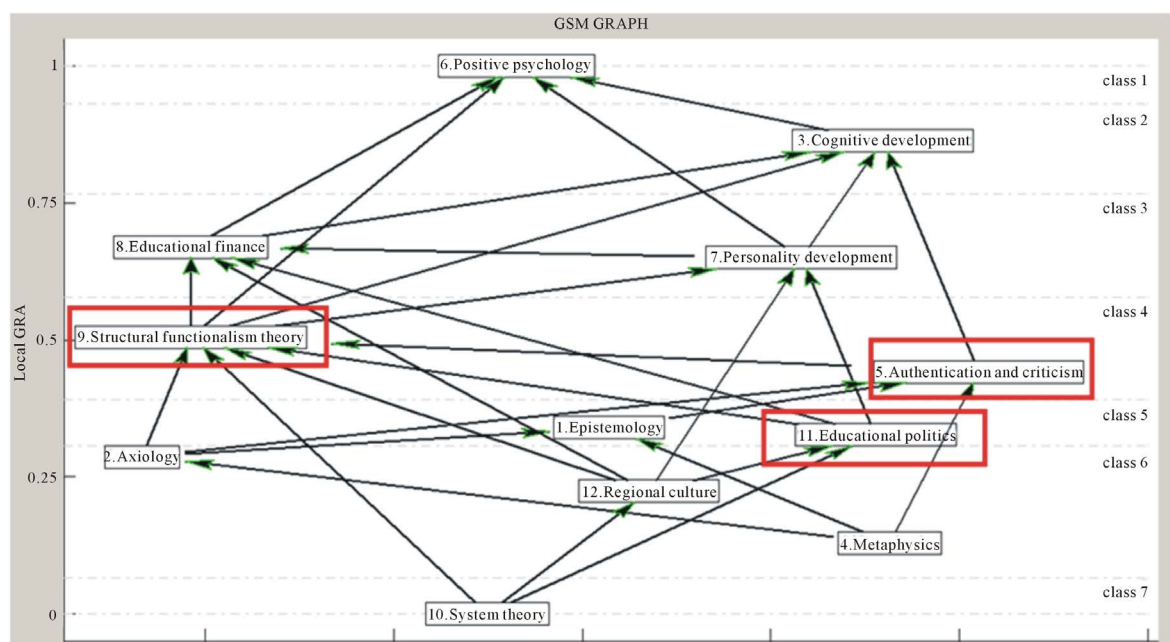
Figure 4. GSM diagram of educational student and researcher (full form).

It is worth mentioning that, the difference between simplified form and complicated (full) form of GSM diagram are: 1) **Figure 3** shows the simplified form structure which is a special case of the complicated form in **Figure 4**, the conceptual structure that can be captured efficiency; 2) In principle, **Figure 4** can be divided into two parts, and the structural functionalism theory is an intermediary, the below of it is leaning to social science and phenomenology; the above of it is about to behavioral science; philosophy, sociology and psychology intertwined from subjective, sociability to one himself; 3) The lower part of figure includes philosophical formalism and social network aggregated to structural functionalism in sociology, both interactive as in- and exterior; the upper part is more individual-oriented psychology.

4.2. Structural Analysis

Firstly, **Table 7** shows that: 1) There are the differences about the cognitive learning between educational undergraduates and researchers, 2) The arrangement of learning difficulties by educational researchers is converged on subjects order: philosophy, psychology, sociology, 3) Learning difficulties ordering by educational undergraduates is psychology, sociology and philosophy.

Secondly, in **Figure 3** and **Figure 4**, the simplified and full forms of kernel GSM diagram show that: 1) The structural functionalism is convergence intersection of metaphysics and system theory, 2) The conceptual structure of locality has meaning or classification characteristics, 3) The conceptual structure of globalization is still difficult and academic knowledge systems, cognitive understanding generate correspondence relation.

Based on the sorting of number value of grey relational analysis, S9 with its *Gamma* = 0.48 in the center (see **Table 3**) and S12 with its *Gamma* = 0.18 (see **Table 5**), This means: 1) The learning outcomes of educational undergraduates and researchers are different; 2) For the people who do not fully understand the concept planning second teaching, the gap between them are large [24]; 3) The second teaching can be implemented the small group cooperative learning, namely those who understand things more support people who understand things in middle level, the weaker ones are implemented the second teaching by the teachers with heterogeneous lesson (the text with multiple design). In order to further understand the learning situation of the second groups, the structure of grey structural modeling is used to analyze the structure.

For the third-year students:

Figure 5 and **Figure 6** show that: 1) The philosophy connect with other conceptual area such as regional culture, cognitive development; 2) The learning difficulty of positive psychology is high, this intention is to promote the depth and breadth of learning, and reveal to enhance the physical, biological to mind of personal character; 3) The distinguishing attribute of knowledge and feelings, knowledge and skills, or knowledge and perception, feelings and skills have improvement space.
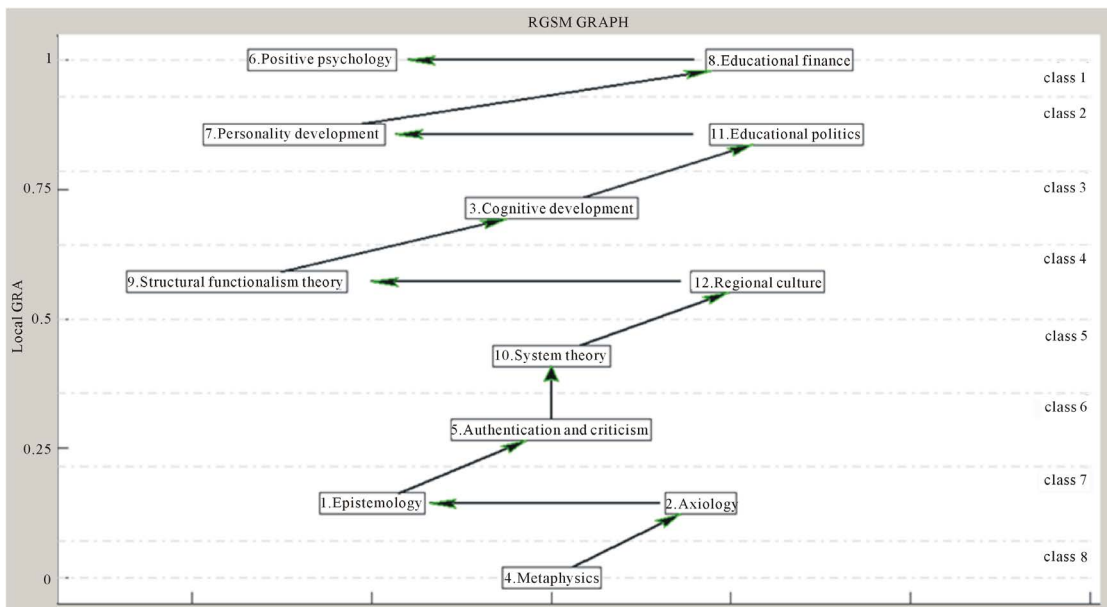

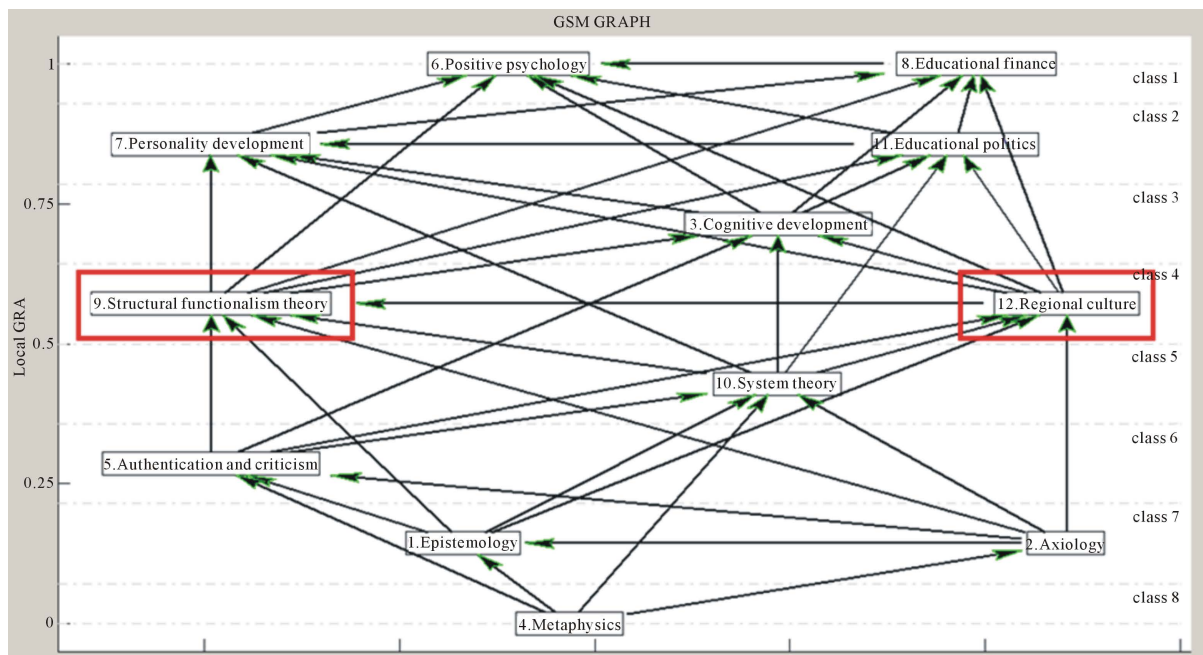**Figure 5.** GSM diagram of educational student (simplified form).

Figure 6. GSM diagram of educational student (full form).

For the syllabus researchers (master degree program):

Figure 7 and **Figure 8** show that: 1) The conceptual compatibleness between sociology and psychology must have logical relationship; 2) Philosophy isolated from the system, only the authentication and criticism element, epistemology, and axiology are associated; and 3) Based on the philosophy is the mother of learning, the relationship between philosophy and sociology is very close, "I am existing, therefore I think," and vice versa, "I am thinking, therefore I exist," reinforcing path such as educational politics supporting on axiology, structural functionalism theory supporting on epistemology, metaphysics must have close relationship with psychology.

Synthesis of descriptions above (**Figures 5-8**), obtained: 1) Educational students and researchers have cognitive gap between them; 2) The special cases in simplified and full forms are possible unified; 3) The vertical direction in full and simplified forms can master the focal points of overall structure, or represent the starting point of learning advance, or making as second teaching, remedial teaching basis, or even on the table in mind the issue of distribution at below or above key points.

4.3. Structural Characterization

Combining Nagai's matrix based structure modeling and graph theory, in order to construct the structural analysis of integrity and visualization (see **Figure 9**), in the first place, it is helpful for communication and diagnostic, in the second place, according to the strategic plan the action is performed. Next, to compare and grasp the knowledge structure development of introduction to education between educational undergraduates and researchers based on the differences of number value, one place **Figure 5**, **Figure 6** and **Figure 9** are the reference basis, the other place, the academic development structure in correspond to the professional consensus and effective academic norms is aimed at.

Finally, the point concepts of convergence and differentiation are: 1) The place where the conversion and transformation of knowledge hierarchy structure exist, 2) Teaching and learning are possible difficulties, 3) Making every effort for the same class with heterogeneous structure.

5. Conclusions

"For great education plan, teacher must be centered". Introduction to education is not only one of the educational compulsory subjects, but also is a subject of structural and scientific cognitive education. With the broad scope and the main academic knowledge system of higher education, teaching and learning quality have become
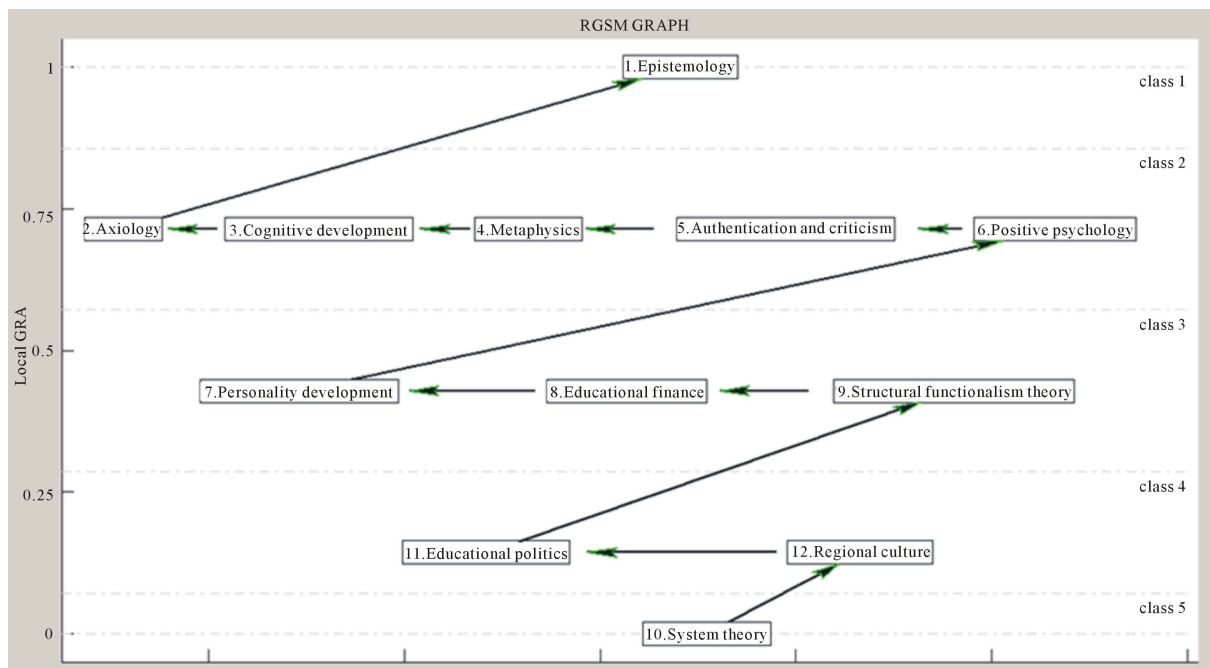
Figure 7. GSM diagram of educational researcher (simplified form).
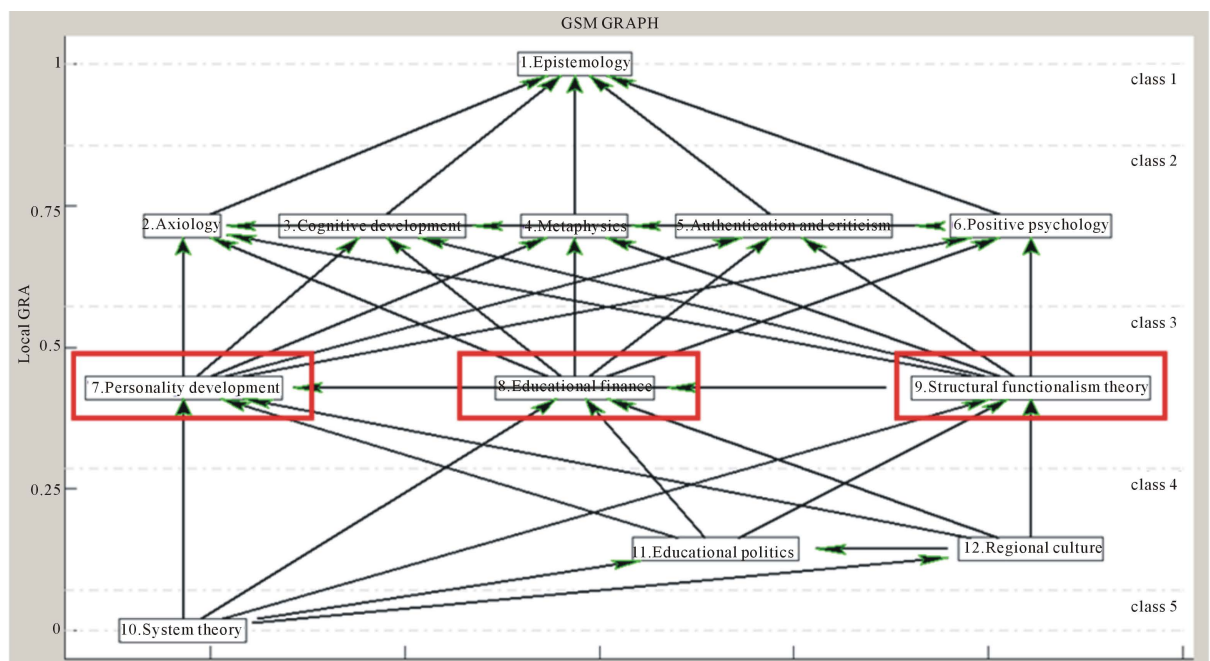

Figure 8. GSM diagram of educational researcher (full form).

the focus, so it takes the learning outcomes of the third year undergraduate educational students and master degree program researchers in primary education course to analyze for understanding. Aggregated by the concepts, the questionnaire respondents have been processed by Nagai's grey relational analysis program, ROSE 2 software, and technique as indicated in **Tables 3-6** and **Figures 2-9**.

Based on the systematic instructional design principles, the starting behavior of learners are based on the same class with heterogeneous structure teaching on classroom, effective control, diagnose this starting behavior is imperative. The conclusion from **Figures 5-8**: 1) The thinking pattern is the inductive and deductive cycles, 2)
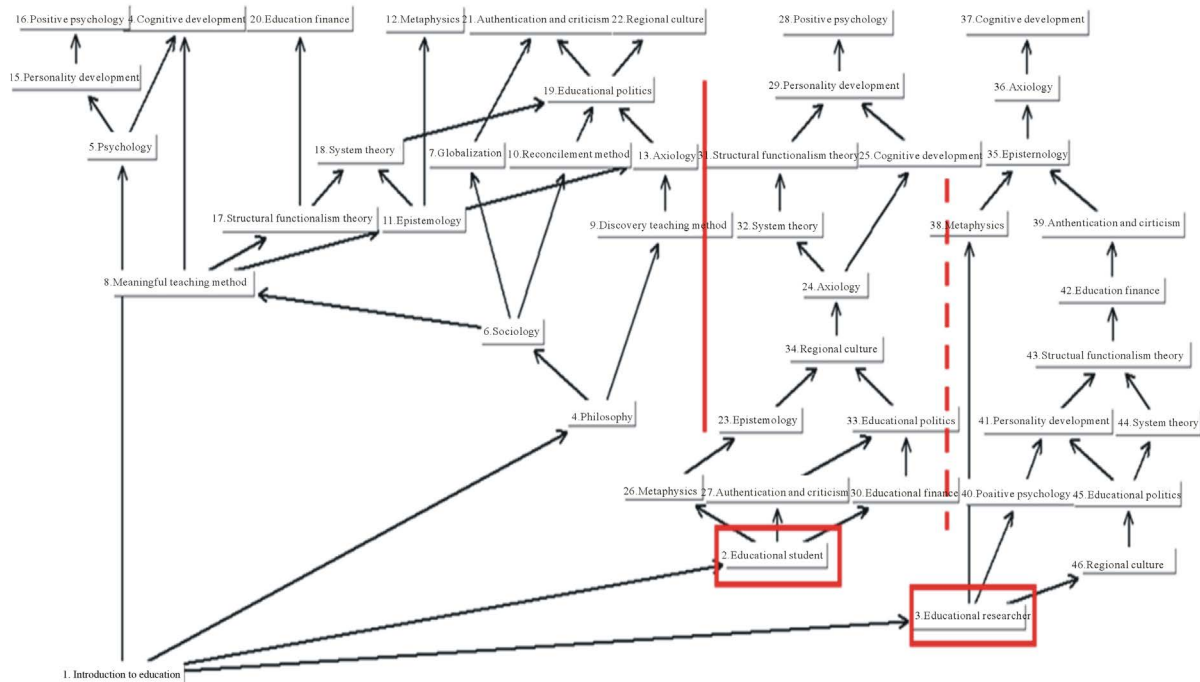
Figure 9. MSM diagram of introduction to education.

The detail concepts converge in inductive point, from a point it can be deduced for expanding differentiation, and spiraling to construct knowledge system; 3) The focal point of directed hierarchical structure can be considered the diagnosis, grouping, evaluation, and making every effort of second education.

Finally, since the 1970s, academic education and professional development have become the norm, enhancing the educational research methodology and research method has become necessary. The combination mathematical tools with professional consensus on education in research are that the data collection and calculation, interpretation of knowledge structure and logical implication, along with systemization and digitalizing to achieve the purpose of prediction and control. Now, the use of mathematical tools to construct the context of situation of confidence interval; using Rough set to reveal the concepts in core, using LGRA to select adjacent learners and construct GSM, MSM combining with graph theory to characterize and visualize the multiple sets structure, it can be seen the following benefits: 1) Scientifically and diagnostically demo learning results between the two groups, 2) Divergence and gap emerges in structural analysis, 3) Structural analysis displays in quantization and visualization.

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