Physical Review & Research International
3(4): 702-712, 2013

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Solitary Wave Solutions of Schrödinger Equation by Laplace−Adomian Decomposition Method

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Authors' contributions

Author RDP designated the study and performed the basic analysis of the problem. Author AK calculated the results and draft the manuscript. All authors read and approved the final manuscript.

Research Article

Received 22nd March 2013
Accepted 26th July 2013
Published 13th August 2013

ABSTRACT

In this paper we combined the Laplace Transform with Adomian Decomposition Method (ADM) and presented an approach for solving non linear coupled and non coupled Schrödinger equation, with initial conditions. It is shown that the method does not need linearization, weak nonlinearity assumptions or perturbation theory to obtain analytical solutions.

Keywords: Laplace−adomian decomposition method; laplace transforms; nonlinear schrödinger equation.

AMS Classification: 35 J 20, 35 J 35.

1. INTRODUCTION

Systems of partial differential equations attracted much attention in a variety of applied sciences and the essential features of these systems are of wide applicability. These systems were formally derived to describe wave propagation in the shallow water [1−5], and

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to examine the chemical reaction-diffusion model of Brusselator type [4−6] and the method of characteristics. The Riemann invariants and Adomian method [6] are the commonly used methods to obtain analytical solutions.

In this work, we extended Laplace Decomposition Method introduced by Khuri [7,8] modified by Agadjanov [9] to study the Non Linear Schrödinger Equation. This technique basically illustrates how the Laplace transform is used to approximate the solutions of the nonlinear partial differential equations by modifying the decomposition method [10-13,14,15].

2. LAPLACE−ADOMIAN DECOMPOSITION METHOD

In this section, we present a Laplace−Adomian Decomposition Method (LADM) for solving the partial differential equations written in an operator form, i.e.

$$\begin{aligned} L_t u + R_1(u,v) + N_1(u,v) &= f_1 \\ L_t v + R_2(u,v) + N_2(u,v) &= f_2 \end{aligned} \tag{1}$$

with initial data

$$\begin{aligned} u(x,0) &= g_1(x) \\ v(x,0) &= g_2(x) \end{aligned} \tag{2}$$

where, $L_t = \frac{\partial}{\partial t}$ is considered as a first-order partial differential operator, R_1, R_2 and N_1, N_2 are linear and nonlinear operators, respectively, and f_1 and f_2 are source terms. The method consists of first applying the Laplace transform to both side of equations in system (1) and then by using initial conditions (2), we have,

$$\begin{cases} L(L_t u) + L(R_1(u,v)) + L(N_1(u,v)) = L(f_1) \\ L(L_t(v)) + L(R_2(u,v)) + L(N_2(u,v)) = L(f_2) \end{cases} \tag{3}$$

Using the differentiation property of Laplace transform, we get

$$\begin{cases} L(u) = \frac{g_1(x)}{p} + \frac{L(f_1)}{p} - \frac{1}{p}\left[L(R_1(u,v)) + L(N_1(u,v))\right] \\ L(v) = \frac{g_2(x)}{p} + \frac{L(f_2)}{p} - \frac{1}{p}\left[L(R_2(u,v)) + L(N_2(u,v))\right] \end{cases} \tag{4}$$

The LADM defines the solutions *u*(*x, t*) and *v*(*x, t*) by the infinite series

$$u(x,t) = \sum_{n=0}^{\infty} u_n,\; v(x,t) = \sum_{n=0}^{\infty} v_n \tag{5}$$

The nonlinear terms N_1, N_2 are usually represented by the infinite series of the so-called Adomian polynomials [16] *i.e.*

$$N_1(x,t)=\sum_{n=0}^{\infty}A_n$$
$$N_2(x,t)=\sum_{n=0}^{\infty}B_n \tag{6}$$

The Adomian polynomials can be generated for all forms of nonlinearity. They are determined by the following relations:

$$A_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N_1\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0} \quad , \quad B_n=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left(N_2\sum_{i=0}^{\infty}\lambda^i v_i\right)\right]_{\lambda=0} \tag{7}$$

Substituting (5) and (6) into (4), gives

$$\begin{cases} L\left(\sum_{n=0}^{\infty}u_n\right)=\frac{g_1(x)}{p}+\frac{L(f_1)}{p}-\frac{1}{p}\left[L\left(R_1\left[\left(\sum_{n=0}^{\infty}u_n\right),\left(\sum_{n=0}^{\infty}v_n\right)\right]\right)+L\left(\sum_{n=0}^{\infty}A_n\right)\right] \\ L\left(\sum_{n=0}^{\infty}v_n\right)=\frac{g_2(x)}{p}+\frac{L(f_2)}{p}-\frac{1}{p}\left[L\left(R_2\left[\left(\sum_{n=0}^{\infty}u_n\right),\left(\sum_{n=0}^{\infty}v_n\right)\right]\right)+L\left(\sum_{n=0}^{\infty}B_n\right)\right] \end{cases} \tag{8}$$

Applying the linearity of the Laplace transform, we define the following recursive formula

$$\begin{cases} L(u_0)=\frac{g_1(x)}{p}+\frac{L(f_1)}{p} \\ L(v_0)=\frac{g_2(x)}{p}+\frac{L(f_2)}{p} \end{cases} \tag{9}$$

$$\begin{cases} L(u_1)=-\frac{1}{p}L\left[\left(R_1\left(u_{0,}v_0\right)\right)\right]-\frac{1}{p}L[A_0] \\ L(v_1)=-\frac{1}{p}L\left[\left(R_2\left(u_{0,}v_0\right)\right)\right]-\frac{1}{p}L[B_0] \end{cases} \tag{10}$$

In general, for $k \geq 1$, the recursive relations are given by

$$\begin{cases} L(u_{k+1})=-\frac{1}{p}L\left[R_1\left(u_{k,}v_k\right)\right]-\frac{1}{p}L[A_k] \\ L(v_{k+1})=-\frac{1}{p}L\left[R_2\left(u_{k,}v_k\right)\right]-\frac{1}{p}L[B_k] \end{cases} \tag{11}$$

Applying the inverse Laplace transform, we can evaluate u_k and v_k ($k \geq 0$). In some cases the exact solution in the closed form may also be obtained.

3. APPLICATIONS

In this section, we use the LADM to solve non linear non coupled and coupled Schrödinger equations.

3.1 Non-linear Schrödinger Equation

The Non-Linear Schrödinger (NLS) equation arises as the envelope of a dispersive wave system which is almost monochromic and weakly nonlinear. The NLS equation has found numerous applications in physics, *e.g.* in the theory of deep-water waves [17] as well as a model for the non-linear pulse propagation in fibers [18]. The modulation of a wave packet in the direction of wave propagation due to dispersive and weakly nonlinear effects is also described by the nonlinear Schrodinger equation [19]. In homogeneous media, when the nonlinear Schrodinger equation has constant coefficients, there are N-soliton solutions and the equation is exactly integrable through the inverse scattering transform technique.

This paper is concerned with the Laplace transform algorithm and the Adomian decomposition method to solve the NLS equation with a new approach.

We consider the NLS equation

$$iE_t + E_{xx} + E|E|^2 = 0 \tag{12}$$

Here *E (x, t)* is the slowly varying envelope of high-frequency field and initial conditions

$$E(x,0) = r\sec h(kx)e^{iqx}$$

Where $r^2 = 2k^2$ and q are arbitrary constants.
Taking the Laplace transform on both sides of Equation (12) and initial conditions and using the differentiation property of Laplace transform we get

$$L(iE_t) + L(E_{xx}) + L(|E|^2 E) = 0$$

$$L(E) = \frac{r\sec h(kx)e^{iqx}}{p} + i\frac{L(E_{xx} + |E|^2 E)}{p}$$

The LADM defines the solutions series $E(x,t)$

$$E(x,t) = \sum_{n=0}^{\infty} E_n$$

Applying inverse Laplace transform we get

$$E(x,t) = r\sec h(kx)e^{iqx} + iL^{-1}\left[\frac{1}{p}L\left(E_{xx} + |E|^2 E\right)\right]$$

$$\sum_{n=0}^{\infty} E_n(x,t) = r\sec h(kx)e^{iqx} + iL^{-1}\left[\frac{1}{p}L\left(E_{nxx} + \sum_{n=0}^{\infty} B_n(E)\right)\right] \quad (13)$$

In above Equation $\sum_{n=0}^{\infty} B_n(E)$ is Adomian polynomials that represent nonlinear terms *i.e.*

$$\sum_{n=0}^{\infty} B_n(E) = E|E|^2$$

The few components of the Adomian polynomials are given as follow

$$E_{n+1}(x,t) = L^{-1}\left[\frac{1}{p}L\left(E_{nxx} + \sum_{n=0}^{\infty} B_n(E)\right)\right]$$

Where $n \geq 0$

$$E_1(x,t) = L^{-1}\left[\frac{1}{p}L\left(E_{oxx} + \sum_{n=0}^{\infty} B_0(E)\right)\right]$$

Where

$$E_{oxx} = \frac{\partial^2 E_0}{\partial x^2} = r\left[\left(k^2 - q^2\right)\sec h(kx) - 2k^2\sec h(kx) - 2kqi\sec h(kx)\tanh(kx)e^{iqx}\right]$$

$$E_o = r\sec h(kx)e^{iqx}$$

$$B_o(E) = E_0|E_0|^2 = r^3\sec h^3(kx)e^{iqx}$$

Then

$$\begin{aligned} E_1(x,t) = rt\Big[&\left(k^2 - q^2\right)\sec h(kx)\cos(kx) + 2kq\sec h(kx)\tanh(kx)\sin(kx) + \\ &i\left\{\left(k^2 - q^2\right)\sec h(kx)\sin(kx) - 2kq\sec h(kx)\tanh(kx)\cos(kx)\right\}\Big] \end{aligned} \quad (14)$$

and

$$E_2(x,t) = L^{-1}\left[\frac{1}{p}L\left(E_{1xx} + \sum_{n=0}^{\infty} B_1(E)\right)\right] \quad (15)$$

Where

$$E_{1xx} = \frac{\partial^2 E_1}{\partial x^2} \qquad B_1(E) = E_0|E_1|^2 + E_1|E_0|^2$$

Then equation (15)

$$E_2(x,t) = r\frac{t^2}{2}\Big[\left(k^2 - q^2\right)^2 \sec h(kx)\cos(qx) + 4k^2q^2 \sec h^3(kx)\cos(qx) -$$
$$4k^2q^2 \sec h(qx)\tanh^2(kx)\cos(qx) +$$
$$2kq\left(2k^2q^2 \sec h(kx)\tanh^3(kx)\sin(qx) - 3k^3 \sec h^3(kx)\tanh(kx)\sin(qx)\right)\Big]$$
$$+ir\left[\left\{\frac{t^2}{2}\left(k^2 - q^2\right)^2 \sec h(kx)\sin(qx) + 4k^2q^2 \sec h^3(kx)\sin(qx) -\right.\right.$$
$$4k^2q^2 \sec h(qx)\tanh^2(kx)\sin(qx) - 2kq2k^2q^2 \sec h(kx)\tanh^3(kx)\cos(qx)$$
$$\left.\left.+3k^3 \sec h^3(kx)\tanh(kx)\cos(qx)\right\}\right] + r\frac{t^3}{3}\left\{2k^2\left(k^2 - q^2\right)\sec h^3(kx)\cos(qx)\right.$$
$$\left.+4qk^3 \sec h^3(kx)\tanh(kx)\sin(qx)\right\} + ri\frac{t^3}{3}\left\{2k^2\left(k^2 - q^2\right)\sec h^3(kx)\sin(qx) -\right.$$
$$\left.4qk^3 \sec h^3(kx)\tanh(kx)\cos(qx)\right\}$$

The other components of the decomposition series can also be determined in a similar way. Substituting these values into equation (13); we can obtain the expression of E(x, t) which is in a Taylor series, then the closed form solutions yield as follows

$$E(x,t) = r\sec h(kx - t\omega)\exp i(qx - \Phi t)$$

Where $\omega = 2kq,\ \Phi = q^2 - k^2$

The solitary wave solution or time evolution of the nonlinear equation up to the second component $E_2(x,t)$ is given in Fig. 1. and the same is compared with the results obtained by the variational method (Fig. 1a) [20].This result can be verified through substitution and it is the same as the exact solution [16].

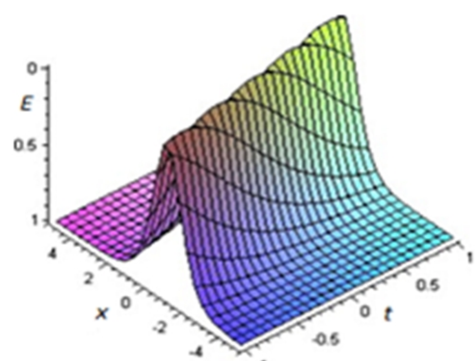

Fig. 1. Exact solitary wave solution of $E_2(x,t)$ with fixed value of k =1/2 for different values of time

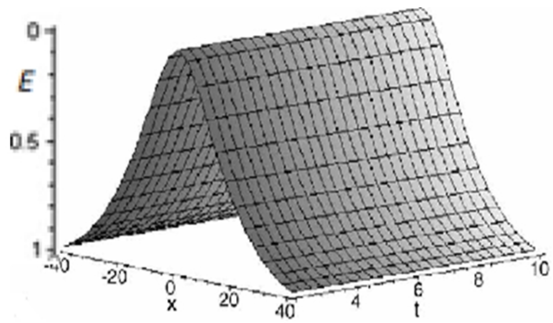

Fig. 1a. Time evolution of the Nonlinear equation, Coefficient E_2 (x,t) by the variational approach for k =1

3.2 Non-Linear Coupled Schrödinger Equation

At the classical level, a set of coupled nonlinear wave equations describes the interaction between high-frequency (*e.g.* Langmuir waves) and low-frequency (*e.g.* ion-acoustic) waves [21]. Since then, this system has been the subject of a large number of studies for both physical and mathematical reasons. Physically, the wave-wave interaction or the wave collisions are common phenomena in science and engineering for both solitary and non-solitary waves. Mathematically solitary wave collision is a major branch of nonlinear wave interaction in ionic media. An example of the model for wave-wave interaction is the coupled 1D nonlinear Schrödinger equation (CNLS), i.e.

$$\begin{aligned} &iE_t + E_{xx} - \eta E = 0 \\ &\eta_{tt} - \eta_{xx} - \left(|E|^2\right)_{xx} = 0 \end{aligned} \tag{16}$$

Where E is the envelope of the high-frequency electric field, η is the plasma density measured from its equilibrium value and initial condition

$$E(x,0) = r\sec h(qx)e^{ikx}$$

$$\eta(x,0) = s - \frac{r^2}{2}\sec h^2(qx)$$

$$\eta_t(x,0) = 2kqr^3\sec h^2(qx)\tanh(qx)$$

Where, $r^2 = 2q^2$, *k, s* are arbitrary constants. Performing Laplace−Adomian decomposition of equation (16) then, by using the differentiation property of Laplace transform and initial conditions gives

$$E(x,t) = E(x,0) + iL^{-1}\left[\frac{1}{p^2}L\left(E_{xx} - \eta E\right)\right]$$

$$\eta(x,t) = \eta(x,0) + t\eta_t(x,0) + L^{-1}\left[\frac{1}{p^2}L\left(\eta_{xx} + \left(|E|^2\right)_{xx}\right)\right]$$

The LADM defines the solutions series $E(x,t)$ $\eta(x,t)$ as

$$E(x,t)=\sum_{n=0}^{\infty}E_n\ ;\ \eta(x,t)=\sum_{n=0}^{\infty}\eta_n \tag{17}$$

$$E_{n+1}=L^{-1}\left[\frac{1}{p^2}L\left(E_{nxx}-\sum_{n=0}^{\infty}A_n(\eta,E)\right)\right]$$

$$\eta_{n+1}=L^{-1}\left[\frac{1}{p^2}L\left(\eta_{nxx}+\sum_{n=0}^{\infty}B_n(E)\right)\right]$$

$$\sum_{n=0}^{\infty}A_n(\eta,E)=\eta E\ ,\ \sum_{n=0}^{\infty}B_n(E)=\left(|E|^2\right)_{xx}$$

Where is Adomian polynomials that represent nonlinear terms.
The few components of the Adomian polynomials are given as follow

$$A_0(\eta,E)=\eta_0E_0 \qquad B_0(E)=\left(|E|^2\right)_{0xx}=\left(E_0\overline{E_0}\right)_{xx}$$

$$A_1(\eta,E)=\eta_0E_1+\eta_1E_0 \quad , \quad B_1(E)=\left(E_0\overline{E_1}\right)_{xx}+\left(E_1\overline{E_0}\right)_{xx}$$

Then

$$E_0=r\sec h(qx)\cos(kx)+ir\sec h(qx)\cos(kx)$$

$$n_0=s-q^2\sec h^2(qx)+4tkq^3\sec h^2(qx)\tanh(qx)$$

$$\begin{aligned}E_1&=L^{-1}\left[\frac{1}{p^2}L\left(E_{0xx}-A_0(\eta,E)\right)\right]\\&=rt\ \left\{\left(q^2-k^2-s\right)\sec h(qx)\cos(kx)-q^3\sec h^3(qx)\cos(kx)+2kq\sec h(qx)\tanh(qx)\sin(kx)\right\}\\&\quad+irt\left\{\left(q^2-k^2-s\right)\sec h(qx)\sin(kx)-q^3\sec h^3(qx)\sin(kx)-2kq\sec h(qx)\tanh(qx)\cos(kx)\right\}\\&\quad+\frac{t^2}{2}4kq^3r\sec h^3(qx)\tanh(qx)\cos(kx)+i\frac{t^2}{2}4kq^3r\sec h^3(qx)\tanh(qx)\cos(kx)\end{aligned}$$

$$\begin{aligned}\eta_1&=L^{-1}\left[\frac{1}{p^2}L\left(\eta_{0xx}+B_0(E)\right)\right]\\&=\frac{t^2}{2}\left(3q^2r^2\tanh^2(qx)\sec h^2(qx)-\frac{3q^2r^2\sec h^2(qx)\left(1-\tanh^2(qx)\right)}{2}\right)\\&\quad-\frac{t^3}{3}\left(\frac{16q^3r^2\sec h^2(qx)\left(1-\tanh^2(qx)\right)\tanh(qx)}{2}-\frac{8q^3r^2\sec h^2(qx)\tanh^3(qx)}{2}\right)\end{aligned}$$

$$E_2(x,t) = -iL^{-1}\left[\frac{1}{p}L\left(E_{1xx} - A_1(\eta, E)\right)\right]$$
$$= rt\ \left\{\left(q^2-k^2-s\right)\sec h(qx)\cos(kx) - q^3\sec h^3(qx)\cos(kx) + 2kq\sec h(qx)\tanh(qx)\sin(kx)\right\}$$
$$+irt\left\{\left(q^2-k^2-s\right)\sec h(qx)\sin(kx) - q^3\sec h^3(qx)\sin(kx) - 2kq\sec h(qx)\tanh(qx)\cos(kx)\right\}$$
$$+\frac{t^2}{2}4kq^3r\sec h^3(qx)\tanh(qx)\cos(kx) + i\left\{\frac{t^2}{2}4kq^3r\sec h^3(qx)\tanh(qx)\cos(kx)+\right.$$
$$\left.+48\sqrt{2}k^3t^2\sec h^2(qx)\tanh(qx)\left(4-9\tanh^2(qx)\right)\left(1-3\tanh^2(qx)\right)\right\}$$
$$-288ik^5t^2\sec h^2(kx)\tanh(kx)\left(1-3\tanh^2(kx)\right)^2 -$$
$$\frac{\sqrt{2}k^2t^2}{3}i\left[216k^3\sec h^2(qx)\tanh^3(qx) - 24k\sec h^2(kx)\tanh(kx)\right]$$

$$\eta_2 = L^{-1}\left[\frac{1}{p^2}L\left(\eta_{1xx} + B_1(E)\right)\right]$$
$$= \frac{t^2}{2}\left(3q^2r^2\tanh^2(qx)\sec h^2(qx) - \frac{3q^2r^2\sec h^2(qx)\left(1-\tanh^2(qx)\right)}{2}\right)$$
$$-\frac{t^3}{3}\left(\frac{16q^3r^2\sec h^2(qx)\left(1-\tanh^2(qx)\right)\tanh(qx)}{2} - \frac{8q^3r^2\sec h^2(qx)\tanh^3(qx)}{2}\right)$$
$$-6q^4\sec h^5(qx)\{q^4\sec h^3(qx)(1208-1191\cosh(2qx)+120\cosh(4qx)$$
$$-\cosh(6qx)) + 24(i\cosh(qx) + 2q\sinh(qx))\}$$

The other components of the decomposition series can also be determined in a similar way, substituting these values into equation (17); we can obtain the expression of E(x, t) and $\eta(x,t)$ which is in a Taylor series, then the closed form solutions yield as follows

$$E(x,t) = r\sec h(qx+\omega t)\exp[i(kx+\Omega t)]$$
$$\eta(x,t) = s - r^2\sec h^2(qx+\omega t)$$

Where $\omega = -2kq$; $\Omega = -s + q^2 - k^2$

This result can be verified through substitution. It is just the same as the exact solution [22]. Thus, we obtain the solutions of the CNLS (16), which are dark and bright solitary wave solutions. The solitary wave solution or time evolution of the nonlinear equation up to the second component $E_2(x,t)$ is given in Fig. 2. The results are compared with the solution (Fig. 2a) derived by the variational approach for the same equation [20].

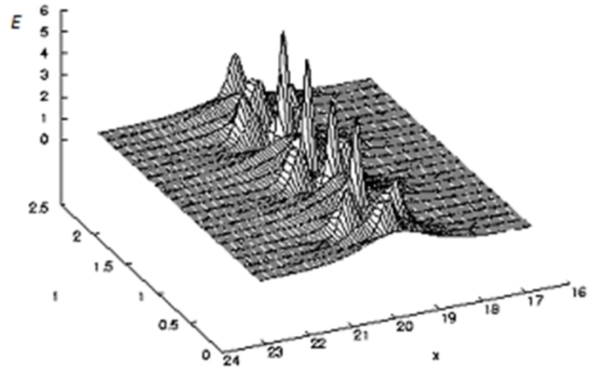

Fig. 2. Exact solitary wave solution of $E_2(x,t)$ with fixed value of k =1/2 for different values of time

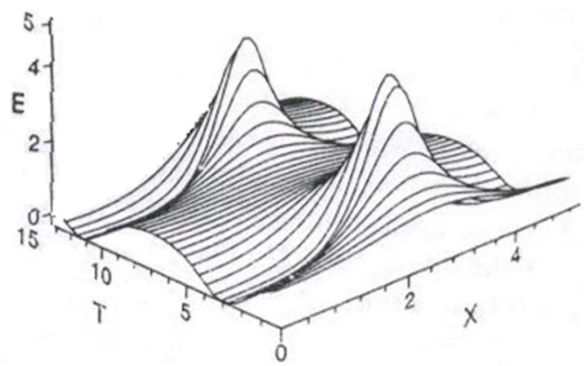

Fig. 2a. Time evolution of the Nonlinear equation by the variational approach for k =1

CONCLUSIONS

In this paper, Laplace−Adomian Decomposition Method was employed successfully for solving nonlinear coupled and non coupled Schrodinger equation. Using this method the problem may be solved without any discretization of variables, therefore, it is not affected by computation round off errors and one does not face the necessity of using large computer memory and time. This method provides a solution of the problem in a closed form while the mesh point techniques only provide the approximation at mesh points. This method is also useful for finding an accurate approximation of the exact solution.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Debnath L. Nonlinear Partial Differential Equations for Scientists and Engineers, Birkhauser, Boston; 1997.
2. Logan JD. An Introduction to Nonlinear Partial Differential Equations, Wiley-Interscience, New York; 1994.

3. Aminikhah, Hossein, Farshid Mehrdoust, Ali Jamalian. A New Efficient Method for Nonlinear Fisher-Type Equations. Journal of Applied Mathematics 2012 (2012), Article ID 586454, 18 pages.
4. Lubich C, Ostermann A. Multigrid dynamic interaction for parabolic equations, BIT. 1987;27:216-234.
5. Vandewalle S, Piessens R. Numerical experiments with nonlinear multigrid waveform relaxation on a parallel processor, Appl. Numer. Math. 1991;8:149-161.
6. Adomian G. The diffusion-Brusselator equation, Comput. Math. Appl. 29 (1995) 1-3.
7. Khuri SA. A Laplace decomposition algorithm applied to class of nonlinear differential equations, J Math. Appl. 2001;4:141-155.
8. Khuri SA. A new approach to Bratu's problem, Appl. Math. Comput. 2004;147:131-136.
9. Elcin Yusufoglu (Agadjanov), Numerical solution of Duffing equation by Laplace decomposition algorithm, Appl. Math. Comput. 2006;177:572-580.
10. Nasser S. Elgazery, Numerical solution for the Falkner-Skan equation, Chaos, Solitons and Fractals. 2008;35:738-746.
11. Wazwaz AM. Partial differential equations methods and applications, Netherland Balkema Pub.; 2002.
12. Wazwaz AM. The variational iteration method for solving linear and nonlinear systems of PDEs, Computers. Math. Appl. 2007;54:895-902.
13. Hussain M, Khan M. Modified Laplace decomposition method, Appl.Math. Sci. 2010;4(36):1769 -83
14. Kumar, Arun, Ram Dayal Pankaj. Laplace Decomposition Method to Study Solitary Wave Solutions of Coupled Nonlinear Partial Differential Equation. ISRN Computational Mathematics. 2012(2012). doi:10.5402/2012/423469
15. Faraz, Naeem, Yasir Khan, Sankar DS. Decomposition-transform method for Fractional Differential Equations. International Journal of Nonlinear Sciences and Numerical Simulation 11. Supplement. 2010;305-310.
16. Arun Kumar. A variational Description of Non-linear Schrödinger Equation International Journal of Computational and Applied Mathematics. 2009;4(2):159–164
17. Kit E, Shemer L. Spatial versions of the Zakharov and Dysthe evolution equations for deepwater gravity waves, J. Fluid Mech. 2002;450:201–205.
18. Anderson D. Variational approach to nonlinear pulse propagation in optical fibers, Phys. Rev. 1983;27(6):3135–3145.
19. Benney DJ, Newell AC. The propagation of nonlinear wave envelopes. J. Math. Phys. 1967;46:133-139.
20. Arun Kumar, Ram Dayal Pankaj. Some solitary and periodic solutions of the non linear wave equations by variational approach. JRAOPS. 2012;11(2):133-139.
21. Fengshun Lu. Couple of the Variational Iteration Method and Legendre Wavelets for Nonlinear Partial Differential Equations. Journal of Applied Mathematics. 2013(2013). Article ID 157956, 11 pages doi.: 10.1155/2013/157956.
22. Wang M, Li X. Extended F-expansion method and periodic wave solutions for the generalized Zakharov equations. Phys Lett A. 2005;343:48–54..

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