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Separation Axioms in Generalized Base Spaces

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Authors' contributions

This work was carried out in collaboration between two authors. Author KE designed the study, wrote the protocol, and managed literature searches. Author FA managed the analyses of the study and wrote the first draft of the manuscript. Both authors read and approved the final manuscript.

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Abstract

The concept of generalized base space is given as a generalization of closure spaces, kernel spaces, topological spaces. The purpose of this paper is to study and investigate some separations axioms in the so-called generalized base spaces. Some characterizations of GBT_i-spaces for $i = 0, 1, 2, 3, 4$ are obtained and some relations among these spaces are established. We study some results concerning separation axioms which are true in general topology, but it is not true in the generalized base spaces.

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1 Introduction

The study of more general structures than that of topological space has taken several directions over the past thirty years. In 1983, Mashhour et al [1] introduced the concept of supra-topology by dropping only the intersection condition. In 1996, Maki [2] studied minimal structures, or shortly m-structures, on a set X, as a collection of subsets of X containing X and the empty set, with no other restriction. Since 1997, Csàszàr has studied topological notions in collections which are closed under unions [3]. They constitute the well-known generalized topologies. As a natural generalization of the above-mentioned structures, in 2011, Csàszàr [4] also introduced the weak structures, which are collections of subsets of X containing the empty set. The weak structure is weaker than each of supra-topology [1], generalized topology [3] and a minimal structure [2, 5]. In addition, the interior and closure operators are introduced within this new context and some important properties of these operations are studied. Many authors characterized some topological notions in such weak structures (see [6]-[12]).

In 2004, Erné [13] proposed, in a completely different context, the so-called base spaces as a generalization of closure spaces, kernel spaces, topological spaces etc. by means of base structures, which are collections of subsets of X with no other restriction.

In 2012, Ávila and Molina [14] defined generalized weak structures (which is the Erné's base structures [13]) as an extension of Csàszàr's weak structures. For them, interior, closure and other related notions are introduced.

In this paper, we aim to continue the study on the notions of base space (or generalized base spaces in this paper) and characterize some of its separation axioms. Through this paper we point out a mistake in [14].

2 Preliminaries

To begin with the simplest definition, we mean by a generalized base structure (base structure in [13] or generalized weak structures in [14]) a subset $\mathcal{M}$ of the power set 2^X of a non-empty set X. The pair $(X, \mathcal{M})$ is called a generalized base space. If we regard $\mathcal{M}$ as an open base structure then, we can regard $\mathcal{M}^c = \{X \setminus B : B \in \mathcal{M}\}$ as a closed base structure. The elements of $\mathcal{M}$ (resp., $\mathcal{M}^c$) are said to be $\mathcal{M}$-open (resp., $\mathcal{M}$-closed).

Definition 2.1. A generalized base structure $\mathcal{M} \in 2^X$ is called:

(i) a weak structure [4] if it satisfies the condition:

(WS) $\phi \in \mathcal{M}$,

(ii) a minimal structure [2] if it is a weak structure and satisfies the condition:

(MS) $X \in \mathcal{M}$;

(iii) a generalized topology [3] if it is a weak structure and satisfies the condition:

(GT) $\forall\ A_i \in \mathcal{M}$ $(i \in I)$ implies $\cup_i A_{i \in I} \in \mathcal{M}$;

(iv) a supra-topology [1](or strong generalized topology in [15]) if it is a generalized topology with $X \in \mathcal{M}$;

(v) a quasi-topology [16, 17] if it is a generalized topology and satisfies the condition:

(QT) if $A, B \in \mathcal{M}$ implies $A \cap B \in \mathcal{M}$.

It is obvious from the definition that each topology, quasi-topology, supra-topology, generalized topology, minimal structure and weak structure are generalized base structures.

As in the usual setting of topology, an $(\mathcal{M}_1, \mathcal{M}_2)$-*continuous map* [13] between generalized base spaces $(X, \mathcal{M}_1)$ and $(Y, \mathcal{M}_2)$ is a function $f : X \longrightarrow Y$ with $f^{\leftarrow}(G) \in \mathcal{M}_1$ if $G \in \mathcal{M}_2$. Where, $f^{\leftarrow}(G)$ designates preimage of G under the map f.

The category of all generalized base spaces as objects and all $(\mathcal{M}_1, \mathcal{M}_2)$-continuous maps as morphisms will be denoted by **GBS**.

A generalized base space $(X, \mathcal{M})$ is said to be GBT_0 [13] if for any pair of distinct points $x, y \in X$, there exists $B \in \mathcal{M}$ with $x \in B \not\ni y$ or $y \in B \not\ni x$.

For a generalized base structure $\mathcal{M} \subset 2^X$, the generalized interior and closure [14, 18] of a subset $A \subseteq X$ are defined by

$$i_{\mathcal{M}}(A) = \bigcup\{G \in \mathcal{M} : G \subset A\}$$

and

$$c_{\mathcal{M}}(A) = \bigcap\{F \in \mathcal{M}^c : A \subset F\}$$

respectively.

Lemma 2.1. *[14, 18] The operation* $i_{\mathcal{M}} : 2^X \longrightarrow 2^X$ *fulfils*

(1) $A \subseteq B \subseteq X$ *implies* $i_{\mathcal{M}}(A) \subseteq i_{\mathcal{M}}(B)$ *for* $A, B \subseteq X$*;*

(2) $i_{\mathcal{M}}(A) \subseteq A$ *for* $A \subseteq X$*;*

(3) $i_{\mathcal{M}}(i_{\mathcal{M}}(A)) = i_{\mathcal{M}}(A)$ *for* $A \subseteq X$*.*

Lemma 2.2. *[14, 18] For the map* $c_{\mathcal{M}} : 2^X \longrightarrow 2^X$*, we have*

(1) $A \subseteq B \subseteq X$ *implies* $c_{\mathcal{M}}(A) \subseteq c_{\mathcal{M}}(B)$ *for* $A, B \subseteq X$*;*

(2) $A \subseteq c_{\mathcal{M}}(A)$ *for* $A \subseteq X$*;*

(3) $c_{\mathcal{M}}(c_{\mathcal{M}}(A)) = c_{\mathcal{M}}(A)$ *for* $A \subseteq X$*.*

As in general topology, by $A^{'}$ we mean the set of all accumulation points of a subset $A \subseteq X$. i.e., the set of all points $x \in X$ such every $G \in \mathcal{M}$ containing x satisfies $(G \backslash \{x\}) \cap A \neq \phi$.

Proposition 2.1. *[14] Let* $\in \mathcal{M}$ *be a generalized base structure on a non-empty set* X*. The following properties hold:*

(1) If $A \in \mathcal{M}$*, then* $i_{\mathcal{M}}(A) = A$*.*

(2) If $A \in \mathcal{M}^c$*, then* $c_{\mathcal{M}}(A) = A$ *and* $A^{'} \subset A$*.*

Conversely $A = i_{\mathcal{M}}(A)$ does not imply $A \in \mathcal{M}$,$A = c_{\mathcal{M}}(A)$ does not imply $A \in \mathcal{M}^c$ and $A^{'} \subset A$ does not imply $A \in \mathcal{M}^c$:

Proposition 2.2. *[14] Let* $\mathcal{M}$ *be a generalized base structure on a non-empty set* X*. For* $A \subseteq X$*, we have* $A^{'} \cup A \subset c_{\mathcal{M}}(A)$*.*

3 On Generalized Base Spaces

In topological setting, we know that a subset is open if and only if its a neighborhood of each of its points. This fact is not completely true in the generalized base setting. To assert that, we give the following:

Definition 3.1. Let $(X, \mathcal{M})$ be generalized base space. A subset B of X is said to be an $\mathcal{M}$-neighborhood or simply a neighborhood of a point $x \in X$ iff there exists $V \in \mathcal{M}$ such that $x \in V \subset B$.

Proposition 3.1. *If $B \in \mathcal{M}$, then it is a neighborhood of each of its points.*

The converse of the above proposition need not be true.

Example 3.1. *Let $\mathcal{M} = \{\{a\}, \{b\}, \{c\}\}$ be an open base structure on $X = \{a, b, c\}$. It is clear that X is a neighborhood of each of its point but not $\mathcal{M}$-open.*

In [[14], Proposition 9 (4)], it is claimed that $A^{'} \cup A = c_{\mathcal{M}}(A)$, for a subset $A \subseteq X$, where $\mathcal{M}$ is a generalized week structure on X. The first part is correct (see **Proposition 2.2**) but the converse is not true in general. The following is a counterexample.

Example 3.2. *Let $X = \{a, b, c, d\}$ and $\mathcal{M} = \{\{a, b\}, \{b, c\}, \phi\}$. For a subset $A = \{b\}$ one have $c_{\mathcal{M}}(A) = X$ and $A^{'} = \{a, c\}$, so $A \cup A^{'} = \{a, b, c\} \subset X$ and therefore $A^{'} = \{a, b, c\} \neq c_{\mathcal{M}}(A)$.*

The above example shows that $A \cup A^{'} \neq c_{\mathcal{M}}(A)$ in general. Also, we can assert that the set $A \cup A^{'}$ is not closed for some $A \subset X$.

Example 3.3. *Let $X = \{a, b, c\}$ and $\mathcal{M} = \{\{a, b\}, \{a, c\}, \{b, c\}\}$. For a subset $A = \{a, b\}$ one have $A^{'} = \{c\}$ and therefore $A \cup A^{'} = X$ which is not closed since $X^c = \phi \notin \mathcal{M}$.*

Now, we introduce the easily established result:

Proposition 3.2. *For $A, B \subseteq X$, and $(X, \mathcal{M})$ be a generalized base space, then*

(1) $c_{\mathcal{M}}(A) \cup c_{\mathcal{M}}(A) \subset c_{\mathcal{M}}(A \cup B)$.

(2) $A^{'} \cup B^{'} \subset (A \cup B)^{'}$.

Although the converse of **Proposition 3.2** is true in topological setting, it is need not true in generalized base space setting. The following is a counterexample.

Example 3.4. *Let $X = \{a, b, c\}$ and $\mathcal{M} = \{\{a, b\}, \{a, c\}, \{b, c\}, \phi\}$. For $A = \{a\}$ and $B = \{b\}$ one have $c_{\mathcal{M}}(A) = \{a\}$,$c_{\mathcal{M}}(B) = \{b\}$, and $c_{\mathcal{M}}(A \cup B) = c_{\mathcal{M}}\{a, b\} = X$. Also, $A^{'} = \phi = B^{'}$, and $(A \cup B)^{'} = \{a, b\}^{'} = \{c\}$. So*

(1) $c_{\mathcal{M}}(A \cup B) \neq c_{\mathcal{M}}(A) \cup c_{\mathcal{M}}(A)$.

(2) $A^{'} \cup B^{'} \neq (A \cup B)^{'}$.

4 Separation Axioms

As we have seen in the previous sections, the concept of a generalized base space $(X, \mathcal{M})$, without any restrictions, is too general for many purposes. In this section some restrictions, called separation axioms, imposed on generalized base spaces and some of their properties and implications are considered.

Definition 4.1. A generalized base space $(X, \mathcal{M})$ is called:

(1) GBT_1 if for any pair of distinct points $x, y \in X$, there exist $A, B \in \mathcal{M}$ with $x \in A \not\ni y$ and $y \in B \not\ni x$.

(2) GBT_2 if for any pair of distinct points $x, y \in X$, there is a disjoint $\mathcal{M}$-open sets A and B with $x \in A$ and $y \in B$.

Remark 4.1. If $(X, \mathcal{M})$ is GBT_i, then it is GBT_{1-1}, $i = 1, 2$.

Example 4.1. *Let $X = \{a, b, c\}$ and*

(1) $\mathcal{M}_0 = \{\{a\}, \{a, b\}\}$;

(2) $\mathcal{M}_1 = \{\{a, b\}, \{a, c\}, \{b, c\}\}$.

It is clear that:

(1) $(X, \mathcal{M}_0)$ *is* GBT_0 *but not* GBT_1.

(2) $(X, \mathcal{M}_1)$ *is* GBT_1 *but not* GBT_2.

Theorem 4.2. *A generalized base space $(X, \mathcal{M})$ is GBT_0 if and only if for each pair of distinct points $x, y \in X$, $c_{\mathcal{M}}(\{x\}) \neq c_{\mathcal{M}}(\{y\})$.*

Proof. $(\Rightarrow)$ Let $(X, \mathcal{M})$ be GBT_0 and $x \neq y$ in X. Then there exists an $\mathcal{M}$-open set B containing one of them but not the other. Without loss of generality, we assume that $x \in B \not\ni y$. It follows that $(X \setminus B) \in \mathcal{M}^c$ and $y \in (X \setminus B) \not\ni x$. So we have that $c_{\mathcal{M}}(\{y\}) \subset (X \setminus B)$ and therefore $x \notin c_{\mathcal{M}}(\{y\})$. But $x \in c_{\mathcal{M}}(\{x\})$, hence $c_{\mathcal{M}}(\{x\}) \neq c_{\mathcal{M}}(\{y\})$.

$(\Leftarrow)$ Suppose that $c_{\mathcal{M}}(\{x\}) \neq c_{\mathcal{M}}(\{y\})$. If the space $(X, \mathcal{M})$ is not GBT_0, then there would exist $x, y \in X$ with $x \neq y$ such that either

(i) no $U \in \mathcal{M}$ such that $x \in U \not\ni y$ or $y \in U \not\ni x$.

or

(ii) every $\mathcal{M}$-open set in X containing both x and y.

Case (i) implies that $\forall U \in \mathcal{M}$, the $\mathcal{M}$-closed set $(X \setminus U)$ containing both x and y. Case (ii) implies that every $\mathcal{M}$-closed set does not contain x or y. In either case, we have that $c_{\mathcal{M}}(\{x\}) = c_{\mathcal{M}}(\{y\})$, which is a contradiction.

□

Theorem 4.3. *A generalized base space $(X, \mathcal{M})$ is GBT_1 if for any point $x \in X$, $\{x\}$ is closed.*

Proof. Suppose that $\{x\}$ is $\mathcal{M}$-closed for every $x \in X$. Let $x, y \in X$ with $x \neq y$. Then by assumption, $\{x\}^c = X \backslash \{x\}$ is a $\mathcal{M}$-open set containing y but not x. Similarly $\{y\}^c = X \backslash \{y\}$ is a $\mathcal{M}$-open set containing x but not y. Hence, $(X, \mathcal{M})$ is GBT_1. □

Although the converse of **Theorem 4.3** is true in topological and generalized topological setting, it is need not true in generalized base space setting. The following is a counterexample.

Example 4.4. *Let $X = \{a, b, c, d\}$ and $\mathcal{M} = \{\{a\}, \{b\}, \{c\}, \{d\}\}$. Then one can note that $(X, \mathcal{M})$ is GBT_1 but for each $x \in X$, the subset $\{x\}$ is not closed.*

Before providing more important results concerning preserving of separation axioms, we need the following definition:

Definition 4.2. A function $f : (X, \mathcal{M}_1) \longrightarrow (Y, \mathcal{M}_2)$ is said to be $(\mathcal{M}_1, \mathcal{M}_2)$-open if $f^{\rightarrow}(A) \in \mathcal{M}_2$, for any $A \in \mathcal{M}_1$, where $f^{\rightarrow}(A) = \{f(a) : a \in A\}$.

Theorem 4.5. *If $f : (X, \mathcal{M}_1) \longrightarrow (Y, \mathcal{M}_2)$ is an injective and $(\mathcal{M}_1, \mathcal{M}_2)$-open mapping of a GBT_2-space $(X, \mathcal{M}_1)$ onto a generalized base space $(Y, \mathcal{M}_2)$, then $(Y, \mathcal{M}_2)$ is GBT_2.*

Proof. Let a and b by any distinct points in Y. Bijectivity of the mapping f implies that there is a pair of distinct points x and y in X with $x = f(a)$ and $y = f(b)$. Since $(X, \mathcal{M}_1)$ is GBT_2, then there is a disjoint $\mathcal{M}_1$-open sets A and B with $a \in A$ and $b \in B$. Since f is open and injective, then $f(A)$ and $f(B)$ are two disjoint $\mathcal{M}_2$-open sets with $x \in f(A)$ and $y \in f(B)$. Consequently $(Y, \mathcal{M}_2)$ is GBT_2. □

Corollary 4.6. *Let $f : (X, \mathcal{M}_1) \longrightarrow (Y, \mathcal{M}_2)$ be an $(\mathcal{M}_1, \mathcal{M}_2)$-open injective mapping of a GBT_1(resp.,GBT_0)-space $(X, \mathcal{M}_1)$ onto a generalized base space $(Y, \mathcal{M}_2)$. Then $(Y, \mathcal{M}_2)$ is GBT_1(resp.,GBT_0).*

Proof. Follows from **Theorem 4.5**. □

Definition 4.3. A generalized base space $(X, \mathcal{M})$ is said to be regular if for each $\mathcal{M}$-closed set F in X and each point $x \in X$ not in F, there exist two disjoint $\mathcal{M}$-open sets, G and H such that $x \in G$ and $F \subseteq H$.

Theorem 4.7. *If $(X, \mathcal{M})$ is a regular generalized base space, then for each $x \in X$ and each $\mathcal{M}$-open set U containing x, there exists an $\mathcal{M}$-open set G such that $x \in G \subseteq c_{\mathcal{M}}(G) \subseteq U$.*

Proof. Since $U \in \mathcal{M}$, then $X \backslash U$ is an $\mathcal{M}$-closed set not containing x and therefore there exist two disjoint $\mathcal{M}$-open sets G and H such that $x \in G$ and $X \backslash U \subseteq H$. So $G \subseteq X \backslash H \subseteq U$. By **Lemma 2.2** and **Proposition 2.1**, $c_{\mathcal{M}}(G) \subseteq c_{\mathcal{M}}(X \backslash H) = X \backslash H \subseteq U$. Thus $x \in G \subseteq c_{\mathcal{M}}(G) \subseteq U$, as asserted. □

The converse of **Theorem 4.7** is not true in general. Let us consider the following example.

Example 4.8. *Let $X = \{a, b, c, d\}$ and $\mathcal{M} = \{X\}$. Since $c_{\mathcal{M}}(X) = X$, then $x \in c_{\mathcal{M}}(X) \subseteq X$, for each $x \in X$. But $(X, \mathcal{M})$ is not regular, since the only $\mathcal{M}$-open set is X and the only $\mathcal{M}$-closed set is ϕ.*

Proposition 4.1. *Every regular GBT_0-space is GBT_2.*

Proof. The proof is the same as in topological setting. □

Definition 4.4. A generalized base space $(X, \mathcal{M})$ is said to be GBT_3 if and only if it is both regular and GBT_1.

Proposition 4.2. *Every GBT_3 is GBT_2.*

Proof. This is a consequence of **Proposition 4.1**. □

The converse of **Propositions 4.1** and **4.2** is not true in general. Let us consider the following example.

Example 4.9. *Let $X = \{a, b, c\}$ and $\mathcal{M} = \{\{a\}, \{b\}, \{c\}, \{a, b\}\}$. It clear that $(X, \mathcal{M})$ is GBT_2. The collection of $\mathcal{M}$-closed sets is $\mathcal{M}^c = \{\{b, c\}, \{a, c\}, \{a, b\}, \{c\}\}$. It is clear that $(X, \mathcal{M})$ is not regular, because for a $\mathcal{M}$-closed set $\{b, c\}$ which does not contain the point a there is no two disjoint $\mathcal{M}$-open sets containing the $\mathcal{M}$-closed set $\{b, c\}$ and the point a respectively.*

Definition 4.5. A generalized base space $(X, \mathcal{M})$ is said to be normal if for each pair of disjoint $\mathcal{M}$-closed sets A and B, there exist two disjoint $\mathcal{M}$-open sets, G and H such that $A \in G$ and $B \subseteq H$.

It is known that if $\mathcal{M}$ is a generalized topology on X with $X \notin \mathcal{M}$, then the generalized topological space $(X, \mathcal{M})$ is normal [[4],**Proposition 2.1**]. But in the generalized base setting, this fact is not true in general. To asset that we give the following example:

Example 4.10. *Let* $X = \{a, b, c, d\}$ *and* $\mathcal{M} = \{\{a, b, c\}, \{b, c, d\}\}$. *Then the collection of* $\mathcal{M}$*-closed sets is* $\mathcal{M}^c = \{\{d\}, \{a\}\}$. *The generalized base space* $(X, \mathcal{M})$ *is not normal, since the disjoint* $\mathcal{M}$*-closed sets* $\{d\}$ *and* $\{a\}$ *are not contained in two disjoint* $\mathcal{M}$*-open sets.*

Theorem 4.11. *If* $(X, \mathcal{M})$ *is a normal generalized base space, then for each* $\mathcal{M}$*-closed* F *and each* $\mathcal{M}$*-open set* U *containing* F*, there exists an* $\mathcal{M}$*-open set* G *such that* $F \subseteq G \subseteq c_{\mathcal{M}}(G) \subseteq U$.

Proof. Since $U \in \mathcal{M}$, then both $X \backslash U$ and F are disjoint $\mathcal{M}$-closed sets. By normality of $(X, \mathcal{M})$, there exist two disjoint $\mathcal{M}$-open sets G and H such that $F \in G$ and $X \backslash U \subseteq H$. So $G \subseteq X \backslash H \subseteq U$. By **Lemma 2.2** and **Proposition 2.1**, $c_{\mathcal{M}}(G) \subseteq c_{\mathcal{M}}(X \backslash H) = X \backslash H \subseteq U$. Thus $F \subseteq G \subseteq c_{\mathcal{M}}(G) \subseteq U$, as asserted. □

The converse of **Theorem 4.11** is not true in general. Let us consider the following example.

Example 4.12. *Let* $X = \{a, b, c, d\}$ *and* $\mathcal{M} = \{\{a\}, \{b\}, \{c\}, \{d\}, X\}$. *Then the collection of* $\mathcal{M}$*-closed sets is* $\mathcal{M}^c = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \phi\}$. *For* $\phi \subset X$*, there exists an* $\mathcal{M}$*-open set, say* $\{a\}$ *such that* $\phi \subset \{a\} \subset c_{\mathcal{M}}(\{a\}) \subseteq X$. *But* $(X, \mathcal{M})$ *is not normal, since the disjoint* $\mathcal{M}$*-closed sets* ϕ *and* $\{a, b, c\}$ *are not contained in any two disjoint* $\mathcal{M}$*-open sets.*

Definition 4.6. A generalized base space $(X, \mathcal{M})$ is said to be T_4 if and only if it is both normal and GBT_1.

We have seen from the above results, that each GBT_i-space is is GBT_{i-1}-space for $i = 1, 2, 3$. But for $i = 4$, the implication is not true in general, as we can see from the following example:

Example 4.13. *Let* $X = \{a, b, c, d\}$ *and* $\mathcal{M} = \{\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{c, d\}\}$. *The collection of* $\mathcal{M}$*-closed sets is* $\mathcal{M}^c = \{\{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, b\}\}$. *It is clear that* $(X, \mathcal{M})$ *is* T_4 *but it is not regular, because for a* $\mathcal{M}$*-closed set* $\{b, c, d\}$ *which does not contain the point* a *there is no two disjoint* $\mathcal{M}$*-open sets containing the* $\mathcal{M}$*-closed set* $\{b, c, d\}$ *and the point* a *respectively.*

5 Conclusion

It has been observed that the concept of a generalized base space, if unrestricted, is to general for many purpose. In the present work, some facts about generalized base spaces has been studied. Some separations axioms in generalized base spaces has been studied. Also, some characterizations of GBT_i-spaces for $i = 0, 1, 2, 3, 4$ are obtained and some relations among these spaces are established. We studied some results concerning separation axioms which are true in general topology, but it is not true in the generalized base spaces.

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Competing Interests

Authors have declared that no competing interests exist.

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