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Optimum Location of Emergency Services for Weighted Callers

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

This paper is an approach for introducing a mathematical treatment of the problem of finding the optimum location(s) of the emergency service centers concerning the case in which the callers for the service have different degrees of importance (weights) leading to present appropriate algorithm needed to solve it. In the end of the paper we introduce a numerical example to illustrate the steps of the algorithm.

Keywords: Emergency; location; service; optimization; weighted; callers.

1 Introduction

The problem of finding the optimum location(s) for the emergency facility centers is one of the important and dynamic problems in the Operations Research field, several mathematicians gave hand in this problem through non-short period of time [1,2,3,4,5]. Also many applications of this problem in different fields were introduced, for example see [6,7,8,9,10]. One of the important version of this problem is that one was figured out by Zimmermann [11], several sub-versions of this problem were discussed and solved to cover different situations and complications of the problem see [12,13,14,15,16,17,18]. In all of these works the problems were about how to find the best possible location to allocate the centers of emergency services like ambulance, fire stations, military supply or even the home delivery services taking in the consideration different situations of the properties of the candidate places to allocate the centers, and possible route from

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each centers to each served place and also the nature of the parameters of the problem (deterministic, stochastic, fuzzy and etc.).

In all these previous works and treatments of the problem it was assumed that all the served customers or callers for the service have the same priority or the same weight of work which is not covering all the realistic situations in which the expected number of calls or the workload or even the expected seriousness of the calls expected from each caller is different.

The aim of this paper is to solve the problem assuming that the different callers for the service have different priorities, so every caller for the service will be assigned to a suitable weight reflecting its importance or its priority. Assigning the weights makes the problem more realistic but it also makes it more complicated and needs special mathematical treatments, a suggested solution and algorithm with example will be introduced.

2 Problem Definition

Assume that n service center for some emergency service, each center s_j has to be located at one line segment $\overline{A_j B_j}, j \in \{1,2,...n\}$ such that the length of each of $A_j B_j = d_j$ is known. Each of these centers can serve n callers for the service that are located at given Areas (approximately points) $Y_i, i \in \{1,2,...,m\}$ assuming that that distance from each caller Y_i to each of the two endpoints of the segment $\overline{A_j B_j}$ is given and denoted by a_{ij}, b_{ij} respectively. Assume also that routes from each service center s_j to any caller has to be passed through one of the endpoints of $\overline{A_j B_j}$ which means that there are two possible routes from each service center to each caller for the service (see Fig. 1. To respond any call (request) for service from any caller Y_i the service provider has to dispatch the needed serving crew from the nearest service center S_j taking the shortest route to respond the request as fast as possible.

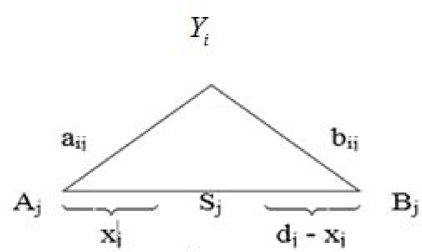

Fig. 1. Possible distances between caller and center

So the distance has to be covered to transport from the service center to the place of the caller is the minimum distance of:

- $x_j + a_{ij}$
- $(d_j - x_j) + b_{ij}$ (where x_j is the distance between the center S_j and the point A_j)

2.1 Unequal callers

To be compatible with the differentiability among the callers for service according to the expected load of work received from each of them or any other factor, the provider of the service has to be more interested in locating the service centers nearer to the caller with higher importance degree than the caller with lower degree. The suggested treatment of this case is depending on assigning a numerical weight to each caller such that the higher weight means higher priority as following:

Assume that for each caller Y_i there is a related positive scalar w_i, this scalar will be called the weight of the caller Y_i.

Each of these weights will be multiplied by the needed distances that will be covered to respond the request of the corresponding caller.

Then the possible distances from each service S_j and the caller Y_i will be:

- $w_i(x_j + a_{ij})$
- $w_i((d_j - x_j) + b_{ij})$

So we can define the minimum weighted distance between the service center S_j and the caller Y_i as:

$$r_{ij}(x_j) = \min \{ w_i(x_j + a_{ij}), w_i[(d_j - x_j) + b_{ij}] \}. \quad (1)$$

As it was mentioned above the concerned services in this paper are emergency services, so every call will be responded by the nearest service center to it, and then we can define the function f_i of the distance needed to answer the caller Y_i by:

$$f_i(x) = \min_{j\in\{1,2,...n\}} \{r_{ij}(x_j)\}, \quad \text{where } x = (x_1, x_2, ..., x_m) \quad (2)$$

In order to guarantee an acceptable level of service the decision maker has dominate the maximum distance between every caller and its related service centers which can be defined as follows:

$$F(x) = \max_{i\in\{1,2,..,m\}} (f_i(x)) \quad (3)$$

The objective of the problem is to find the suitable vector $x = (x_1, x_2, ..., x_n)$ that minimize the function $F(x)$ assuming that the value of $F(x)$ can not exceed given positive scalar α.

The problem can written as:

Definition 1: (P_1)

$$F(x) \longrightarrow \text{Min}$$
$$\text{Subject to} \quad F(x) \leq \alpha,$$
$$0 \leq x_j \leq d_j$$

This problem had to be proved as non-convex, non-differentiable and NP-hard problem, see [5]. To avoid this complications several attempts to solve it by adding special assumptions [11] or by finding numerical approximated (not exact) solutions of it see [12], more generalizations and different versions of this problem also were introduced see [12,13].

In the following we will present ordering assumptions that are needed to solve the problem by effective algorithm.

2.2 The ordering assumptions

Assumption 1: For each $j \in \{1, 2, n\}$ there exists a permutation $\pi = (1^*, 2^*, m^*)$ of the indices $(1, 2, m)$ such that:

$$(w_{i_1^*}a_{i_1^*j}, w_{i_1^*}(a_{i_1^*j}+d_j), w_{i_1^*}b_{i_1^*j}, w_{i_1^*}(b_{i_1^*j}+d_j)) \le$$
$$(w_{i_2^*}a_{i_2^*j}, w_{i_2^*}(a_{i_2^*j}+d_j), w_{i_2^*}b_{i_2^*j}, w_{i_2^*}(b_{i_2^*j}+d_j)) \le$$
$$,\ldots\ldots, \le (w_{i_m^*}a_{i_m^*j}, w_{i_m^*}(a_{i_m^*j}+d_j), w_{i_m^*}b_{i_m^*j}, w_{i_m^*}(b_{i_m^*j}+d_j))$$

Therefore, for each segment, $\overline{A_jB_j}$ there is ordering for the set $\{Y_i: i \in \{1, 2,\ldots m\}\}$ according to the weighted distances between each of them and the end points of the line segment.

This ordering relation guarantees that:

$$\forall x \in [0, d_j],\quad r_{i_1^*j}(x) \le r_{i_2^*j}(x) \le \ldots \le r_{i_m^*j}(x)$$

Remark: This ordering assumption is a generalisation of the hypothesis that was imposed by Zimmermann at [17] which can be considered a particular case of the above ordering assumption and can be obtained by putting all the weights $w_i = 1$.

Definition 2:

For each positive scalar α and each $i \in \{1,2,\ldots,m\}$ and $j \in \{1,2,\ldots,n\}$ the set $V_{ij}(\alpha)$ is defined as:

$$V_{ij}(\alpha) = \{x : x \in [0, d_j] \wedge r_{ij}(x) \le \alpha\},$$

and

$$M(\alpha) = \{x=(x_1, x_2, \ldots x_n): 0 \le x_j \le d_j, F(x) \le \alpha\}$$

The above ordering assumption yields directly the following one:

Assumption 2:

For each j $\in \{1, 2, \ldots, n\}$, there exists a permutation $\pi = (1^*, 2^*, \ldots, m^*)$ of the indices $(1, 2, \ldots, m)$ such that:

For each positive scalar α, $V_{i_1^*j}(\alpha) \supseteq V_{i_2^*j}(\alpha) \supseteq \ldots \supseteq V_{i_m^*j}(\alpha)$

Example:

a. If there are two callers ($m = 2$), with relative weights $w_1 = 2$, $w_2 = 3$ respectively *and one center* ($n = 1$), Let the distances between each caller and each point of the street in which the center will be located are: $a_{1j} = 8$, $b_{1j} = 4$, , $a_{2j} = 6$, $b_{2j} = 5$ respectively and let the distance between the end points of the street is $d_j = 5$, (Fig. 2a),

then, $(w_1 a_{1j}, w_1 (a_{1j}+ d_j), w_1 b_{1j}, w_1 (b_{1j}+ d_j)) = (16, 26, 18, 8)$,

and $(w_2 a_{2j}, w_2 (a_{2j}+ d_j), w_2 b_{2j}, w_1 (b_{2j}+ d_j)) = (18, 33, 30, 15)$

So $r_{1j} = \begin{cases} 16+2x \rightarrow ifx \le 0.5 \\ 18-2x \rightarrow ifx > 0.5 \end{cases}$ and $r_{2j} = \begin{cases} 18+3x \rightarrow ifx \le 2 \\ 30-3x \rightarrow ifx > 2 \end{cases}$ (Fig. 2b)

If α =20 then $V_{1j}(\alpha) = [0,5]$ and $V_{2j}(\alpha) = [0, \frac{2}{3}] \cup [\frac{10}{3}, 5]$ which means $V_{1j}(\alpha) \supseteq V_{2j}(\alpha)$

It is satisfying both the above two ordering assumptions.

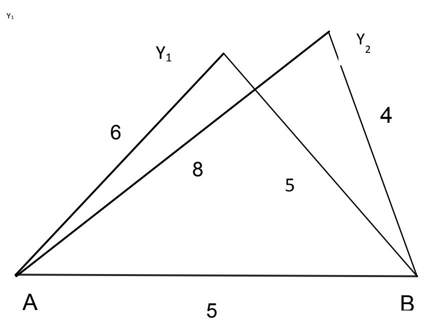

Fig. 2a. Callers that satisfy the ordering assumption

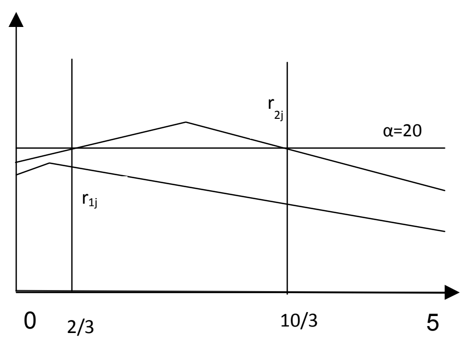

Fig. 2b. $V_{1j}(\alpha) \supseteq V_{2j}(\alpha)$

b. The same like the above example but with different parameters $m = 2$, $w_1 =2$, $a_{1j} = 10$, $b_{1j} = 6$, $w_2 = 3$, $a_{2j} = 6$, $b_{2j} = 5$, $d_j = 10$ (Fig. 3a),

then, $(w_1 a_{1j}, w_1 (a_{1j}+ d_j), w_1 b_{1j}, w_1 (b_{1j}+ d_j)) = (20, 40, 32, 12)$,

and $(w_2 a_{2j}, w_2 (a_{2j}+ d_j), w_2 b_{2j}, w_1 (b_{2j}+ d_j)) = (18, 48, 45, 15)$

So $r_{1j} = \begin{cases} 20+2x \rightarrow ifx \le 3 \\ 18-2x \rightarrow ifx > 3 \end{cases}$ and $r_{2j} = \begin{cases} 18+3x \rightarrow ifx \le 4.5 \\ 45-3x \rightarrow ifx > 4.5 \end{cases}$. (Fig. 3b)

If α =20 then $V_{1j}(\alpha) = [0,6]$ and $V_{2j}(\alpha) = [0, \frac{2}{3}] \cup [\frac{25}{3}, 10]$ which means that neither $V_{1j}(\alpha) \supseteq V_{2j}(\alpha)$ nor $V_{2j}(\alpha) \supseteq V_{1j}(\alpha)$, so it does not satisfy any of the above two ordering assumptions.

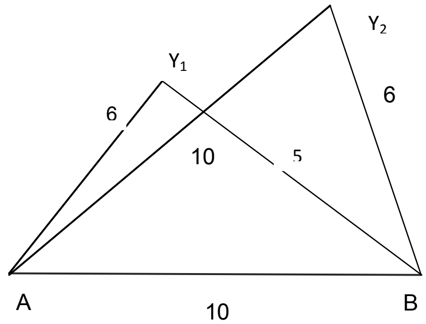

Fig. 3a. Callers that don't satisfy the ordering assumption

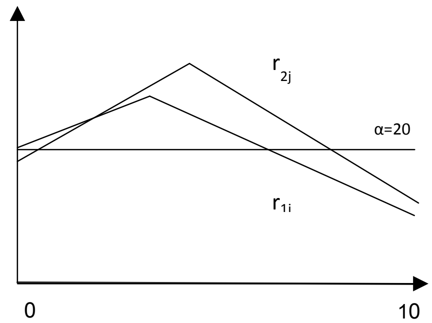

Fig. 3b. Neither $V_{1j}(\alpha)\supseteq V_{2j}(\alpha)$ **nor** $V_{2j}(\alpha)\supseteq V_{1j}(\alpha)$

So we can rewrite problem ($P1$) as follows:

Definition 4 (problem *P*2)

Objective function $\alpha \rightarrow min$
Subject to $M(\alpha) \neq \phi$
$0 \le x_j \le d_j$

3 Mathematical Treatments

In this section, we derive the mathematical results and conclusions leading to the target algorithm which solve the above problem.

3.1 Mathematical conclusions

Here we will perform some needed scientific findings to construct an efficient algorithm to solve the problem (*P2*).

Remark:

In the following, we will restrict our study to the case in which $r_{ij}(0) = w_i a_{ij}$ and $r_{ij}(d_j) = w_i b_{ij}$, nothing will be lost by this restriction because the excluded cases is turning the problem to the case of one route problem which is easier than the case we concern with.

Definition 5

$\forall$ i $\in$ {1, 2, . . . ,m}, $\forall$ j $\in$ {1, 2, . . . , n}, define:
$\delta_{ij} = \min\{r_{ij}(x_j) : 0 \le x_j \le d_j\}$
$\rho_{ij} = \max\{r_{ij}(x_j) : 0 \le x_j \le d_j\}$

Lemma 1

$\forall$ i $\in$ {1, 2, . . . ,m}, $\forall$ j $\in$ {1, 2, . . . , n}, we can show that
1) $\delta_{ij} = \min\{w_i a_{ij}, w_i b_{ij}\}$
2) $\rho_{ij} = ½ (b_{ij} + a_{ij} + d_j)$

Proof

1) Clear and direct
2) ρ_{ij} is the value of the function $r_{ij}(x_j)$ at the point in which the two functions

$f_1(x_j) = w_i(a_{ij}+x_j)$ and $f_2(x_j) = w_i(d_j-x_j+b_{ij})$ *are equal to each other*

Then the value of $x_j = ½ (b_{ij} - a_{ij} + d_j)$

By substituting this value in any of $f_1(x_j)$ *or* $f_2(x_j)$ it is found that $\rho_{ij} = ½ w_i (b_{ij} + a_{ij} + d_j)$

Definition 6

$\forall$ i $\in$ {1, 2, m}, $\forall$ j $\in$ {1, 2, . . . , n}, define:

$$L_{ij} = \frac{\alpha}{w_i} - a_{ij} \quad \text{And} \quad H_{ij} = d_j + b_{ij} - \frac{\alpha}{w_i}.$$

Lemma 2

If $V_{ij}(\alpha) \neq \phi$, then: $L_{ij} \geq 0$ or $H_{ij} \leq d_j$

Proof:

If $V_{ij}(\alpha) \neq \phi$, then $\delta_{ij} \leq \alpha$ which means $w_i a_{ij} \leq \alpha$ or $w_i b_{ij} \leq \alpha$

That means $\frac{\alpha}{w_i} \geq a$ or $\frac{\alpha}{w_i} \geq b$

So $L_{ij} \geq 0$ or $H_{ij} \leq d_j$.

3.2 The possible cases of $V_{ij}(\alpha)$

Now we can conclude directly all the possible case of the set $V_{ij}(\alpha)$ as follows:

Theorem 1: $\forall \alpha \in R^+$ the set $V_{ij}(\alpha)$ will be always has one of the following cases:

- If $\delta_{ij} > \alpha$ then $V_{ij}(\alpha) = \varphi$
- If $\rho_{ij} \leq \alpha$ then $V_{ij}(\alpha) = [0, d_j]$
- If $L_{ij} = 0$ and $H_{ij} > d_j$ then $V_{ij}(\alpha) = \{0\}$
- If $L_{ij} < 0$ and $H_{ij} = d_j$ then $V_{ij}(\alpha) = \{d_j\}$
- If $L_{ij} = 0$ and $H_{ij} = d_j$ then $V_{ij}(\alpha) = \{0, d_j\}$
- If $L_{ij} > 0$ and $H_{ij} > d_j$ then $V_{ij}(\alpha) = [o, L_{ij}]$
- If $L_{ij} < 0$ and $H_{ij} < d_j$ then $V_{ij}(\alpha) = [H_{ij}, d_{ij}]$
- If $L_{ij} > 0$ and $H_{ij} < d_j$ then $V_{ij}(\alpha) = [o, L_{ij}] \cup [H_{ij}, d_{ij}]$

Proof is directly obtained from lemmas 1 and lemma 2.

Lemma 3

(M (α) ≠ Φ) if and only if ∀ i ∈ {1, . . . , m}, , ∃ j(i)∈ {1, . . . , n} such that $V_{ij(i)}(\alpha) \neq \Phi$.

The proof is direct.

Definition 7

∀ i ∈ {1, 2, . . . ,m}, ∀ j ∈ {1, 2, . . . , n}, define:

$P_j(\alpha) = \{i: i \in \{1, \ldots, m\}, V_{ij} \neq \Phi\}$ $\quad V_j(\alpha) = \bigcap_{i \in P_j(\alpha)} V_{ij}(\alpha)$.

Also ∀ i ∈ {1, . . . , m} define:

$\tilde{\alpha}_i = \min \{\alpha: \exists j \in \{1, \ldots, n\}$ such that $V_{ij}(\alpha) \neq \Phi\}$.

Lemma 4 ∀i ∈ {1, . . . , n}, $\tilde{\alpha}_i = \min_{j \in \{1,2,\ldots,n\}} \delta_{ij}$

Proof:

<u>*First*</u>: It is clear that $\forall i \in \{1,2,...,m\}, \exists j^* \in \{1,2,...,n\}$ such that $\delta_{ij^*} = \min_{j \in \{1,2,\ldots,n\}} \delta_{ij}$,

then $V_{ij^*}(\delta_{ij^*}) \neq \phi$

<u>*Second:*</u> If $\alpha < \min_{j \in \{1,2,\ldots,n\}} \delta_{ij}$, then $\forall j \in \{1,2,...,n\}$, $\delta_{ij} > \alpha$,

Which means that $V_{ij}(\alpha) = \Phi$ (theorem 1).

Then $\tilde{\alpha}_i = \min_{j \in \{1,2,\ldots,n\}} \delta_{ij}$

Theorem 2

From the above lemmas we can conclude that:

The optimal solution of the optimization problem **P2** is given by:

$\alpha = \max \{\tilde{\alpha}_i : i = 1, \ldots, m\}$

Using the above theorem and the above cases of $V_{ij}(\alpha)$ taking in consideration the ordering assumptions assumed before, it is obtained that for all j ∈ {1, 2, . . . , n} the set $V_j(\alpha)$ can be calculated with polynomial computational complexity.

So we can now use the following algorithm to solve the problem (P2), the steps of this algorithm are nearly similar to the levels of that algorithm mentioned in [13].

4 Algorithm

We are ready now to introduce the algorithm which can be described as follows:

STEP 1: $\forall$ i $\in$ {1, 2, . . . ,m}, $\forall$ j $\in$ {1, 2, . . . , n}. Input the following parameters:
• The Number of Callers Y_i (m)
• The Number of Emergency Service Centers S_j (n)
• The Spatial Parameters a_{ij}, b_{ij}, w_i, d_j.
• Maximum threshold α^* (optional)
STEP 2: Test the satisfaction of the ordering assumption; if the assumption is satisfied go to Step 3. Else stop and print "the ordering assumption is not satisfied".
STEP 3: $\forall$ i $\in$ {1, 2, . . . ,m}, $\forall$ j $\in$ {1, 2, . . . , n}. Calculate the minimum value
$\delta_{ij} = \min\{w_i a_{ij}, w_i b_{ij}\}$
STEP 4: $\forall$ i $\in$ {1, 2, . . . ,m}, $\forall$ j $\in$ {1, 2, . . . , n}. Calculate the maximum value
$\rho_{ij} = \max\{r_{ij} : 0 \le x_j \le d_j\} = \frac{1}{2} w_i (b_{ij} + a_{ij} + d_j)$
STEP 5: $\forall$ i $\in$ {1, 2, . . . ,m}. Calculate the minimum value
$\tilde{\alpha}_i = \min\{\delta_{ij} : j = 1, \ldots, n\}$
STEP 6: If the maximum threshold α^* is imposed then update $\tilde{\alpha}_i$ $\forall$ i $\in$ {1, 2, . . . , m} as follows : $\tilde{\alpha}_i = \alpha^*$ if $\alpha^* \ge \tilde{\alpha}_i$ and $\tilde{\alpha}_i = \tilde{\alpha}_i$ if $\alpha^* < \tilde{\alpha}_i$
STEP 7: Calculate the maximum threshold value α
$\alpha = \max\{\tilde{\alpha}_i : i = 1, \ldots, m\}$
STEP 8: $\forall$ i $\in$ {1, 2, . . . , m}, $\forall$ j $\in$ {1, 2, . . . , n}. Calculate the following values

$$L_{ij} = \frac{\alpha}{w_i} - a_{ij} \quad \& \quad H_{ij} = d_j + b_{ij} - \frac{\alpha}{w_i}.$$

STEP 9: $\forall$ i $\in$ {1, 2, . . . , m}, $\forall$ j $\in$ {1, 2, . . . , n}. Calculate the set $V_{ij}(\alpha)$, and then calculate the V (α) matrix by applying the investigated cases in the previous section:
STEP 10: $\forall$ i $\in$ {1, 2, . . . ,m}. Determine an index j(i), j $\in$ {1, 2, . . . , n}, such that: Vij (α) $\neq \Phi$. If there is more than one, then break the tie arbitrary.
If $V_{ij}(\alpha) = \Phi$ for each index j , j $\in$ {1, . . . , n}, then **Go to step 12.**
STEP 11: $\forall$ j $\in$ {1, . . . , n} Determine:
$P_j(\alpha) = \{i : i \in \{1, \ldots, m\}, V_{ij}(\alpha) \neq \varphi\}$

$$V_j = \bigcap_{i \in P_j(\alpha)} V_{ij}(\alpha)$$

$$x_j^{opt} \in \begin{cases} \mathrm{V_j} & \text{if} \quad \mathrm{P_j}(\alpha) \neq \varnothing \\ [0, \mathrm{d_j}] & \textit{if} \quad \mathrm{P_j}(\alpha) = \varnothing \end{cases}$$

Go to step 13.
STEP 12: STOP. Print there is no solution of the problem
STEP 13: STOP. Print the solution of the problem is: $(x_1^{opt}, x_2^{opt}, \ldots\ldots\ldots, x_n^{opt})$

5 Numerical Example

To illustrate the steps of the proposed algorithm. Assume that four callers (i.e. m = 4) with relative weights $w_1 = 0.5$, $w_2 = 0.7$, $w_3 = 1$ and $w_4 = 1.5$, assume also that there is three service centers (i.e. n = 3) each of them will be located in one of three street with the same length $d_1 = d_2 = d_3 = 3$. Assume also that there are two routes from each caller to each suggested streets.

Let the following matrix represents the distances from each caller to the end points of each street:

$$(a_{ij}, b_{ij}) = \begin{bmatrix} (1,2) & (6,7) & (6,5) \\ (2,2) & (5,5) & (7,7) \\ (3,4) & (4,4) & (8,9) \\ (4,5) & (3,3) & (4,5) \end{bmatrix}.$$

If the manager assumes that, there is a standard time T, such that the longest distance between any ambulance centre and any caller that will be served by cannot exceed five units of length (i.e. $\alpha^* = 5$).

By checking the elements of the following matrix,

$$(w_i a_{ij}, w_i(a_{ij}+d_j), w_i b_{ij}, w_i(b_{ij}+d_j)) = \begin{bmatrix} (0.5,2,1,2.5) & (3,4.5,3.5,5) & (3,4.5,2.5,4) \\ (1.4,3.5,1.4,3.5) & (3.5,5.6,3.5,5.6) & (4.9,7,4.9,7) \\ (3,6,4,7) & (4,7,4,7) & (8,11,9,12) \\ (6,10.5,7.5,12) & (4.5,9,4.5,9) & (6,10.5,7.5,12) \end{bmatrix}$$

It is found that the ordering assumptions are satisfied.

Applying steps 3, 4, 5 of the algorithm, gives the values of

$$\delta_{ij} = \begin{bmatrix} 0.5 & 3 & 2.5 \\ 1.4 & 3.5 & 4.9 \\ 3 & 4 & 8 \\ 6 & 4.5 & 6 \end{bmatrix} \quad \rho_{ij} = \begin{bmatrix} 1.5 & 4 & 3.5 \\ 2.45 & 4.55 & 5.95 \\ 5 & 5.5 & 10 \\ 9 & 6.75 & 9 \end{bmatrix} \quad \tilde{\alpha}_i = \begin{bmatrix} 0.5 \\ 1.4 \\ 3 \\ 4.5 \end{bmatrix}$$

Step 7 gives the maximum threshold $\alpha = 5$.

Steps 8, 9, 10 of the algorithm, give the values of:

$$L_{ij} = \begin{bmatrix} 9 & 4 & 7 \\ \frac{36}{7} & \frac{15}{7} & \frac{1}{7} \\ 2 & 1 & -3 \\ \frac{-2}{3} & \frac{1}{3} & \frac{-2}{3} \end{bmatrix} \qquad H_{ij} = \begin{bmatrix} -1 & 4 & 20 \\ \frac{-15}{7} & \frac{6}{7} & \frac{20}{7} \\ 2 & 2 & 7 \\ \frac{14}{3} & \frac{8}{3} & \frac{14}{3} \end{bmatrix}$$

And

$$V_{ij}(\alpha) = \begin{bmatrix} [0,3] & [0,3] & [0,3] \\ [0,3] & [0,3] & [0,\frac{1}{7}]\cup[\frac{20}{7},3] \\ [0,3] & [0,1]\cup[2,3] & \Phi \\ \Phi & [0,\frac{1}{3}]\cup[\frac{8}{3},3] & \Phi \end{bmatrix}$$

Steps 11, 12, 13 of the algorithm, give the values of:

$$P_j = \left[\{1,2,3\} \quad \{1,2,3,4\} \quad \{1,2\}\right]$$

Which means that the one centre located in suitable place of street number 2 can serve all the four expected callers

And
$$V_j = \left[[0,3] \quad [0,\frac{1}{3}]\cup[\frac{8}{3},3] \quad [0,\frac{1}{3}]\cup[\frac{20}{3},3]\right]$$

Therefore
$$x^{opt} = \begin{bmatrix} x_1^{opt} \\ x_2^{opt} \\ x_3^{opt} \end{bmatrix} \in \begin{bmatrix} [0,3] \\ [0,\frac{1}{3}]\cup[\frac{8}{3},3] \\ [0,\frac{1}{3}]\cup[\frac{20}{3},3] \end{bmatrix}$$

6 Conclusion

In the presented work we have introduced a mathematical treatment for the optimisation problem of locating the service centres of the emergency service centres including:

- Formulating the question assuming that the customers of the service have different importance degrees (priorities).
- Setting the suitable assumptions that are necessary to solve the problem by the time-wisely useful algorithm
- Deriving the scientific conclusions of the problem.
- Introducing a practical algorithm that solves the problem.
- Presenting a numerical example illustrating the steps of the algorithm.

Competing Interests

Author has declared that no competing interests exist.

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