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Modelling Mother-To-Child HIV Transmission Rate in Nigeria Using an Exponentiated Exponential Inverse Exponential Distribution

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Authors' contributions

This work was carried out in collaboration among all authors. Authors AIA and IBE designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author AAU managed the analyses of the study. Author TGI managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

The act of adding extra parameters into existing distributions for increasing their flexibility or performance is a giant stride in the area of statistical theory and applications. Acquired immune deficiency syndrome (AIDS) is a disease caused by human immunodeficiency virus (HIV) that leads to a progressive deterioration of the immune system. Mother-to-child transmission of HIV is a problem in Nigeria where its rate has been on an increase over the past few years. The Exponentiation family is one of the most efficient methods proposed and studied for introducing skewness and flexibility into continuous probability distributions with a single shape parameter. In this paper, the method of exponentiation has been used to add flexibility to the exponential inverse exponential distribution which results to a new continuous model known as "Exponentiated

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Exponential Inverse Exponential distribution". The properties, application and estimation of parameters of the new distribution using the method of maximum likelihood estimation are presented and discussed in this paper. The new model has been applied to a dataset on the rate of mother-to-child transmission of HIV and the result is being compared among the fitted distributions using some information criteria.

Keywords: Exponential inverse exponential distribution; exponentiation family; properties; maximum likelihood estimation; application.

1. INTRODUCTION

The exponential distribution used in Poisson processes describes the time between events. Many of its applications are carried out in life testing experiments. It has memoryless property with constant failure rate making it unfit for real life situations and then creating a problem in statistical modeling and applications.

In order to make the exponential distribution better, [1] proposed a modified version of the exponential distribution called the inverse exponential distribution which has been studied in some details by [2].

The inverse exponential distribution was found adequate for modeling datasets with inverted bathtub failure rates but it also has a limitation which is its inability to efficiently analyze datasets that are highly skewed (either positively or negatively) [3]. This therefore makes it necessary for introducing skewness and flexibility into the inverse exponential distribution to enable it adequately model heavily skewed datasets.

It is worthy to note that there are many generalizations of the exponential or inverse exponential distribution using differently proposed families of continuous probability distributions and some of these recent studies include the odd Lindley inverse exponential distribution [4], the Exponential Inverse Exponential distribution [5], the Kumaraswamy Inverse Exponential distribution [6], the exponentiated generalized Inverse Exponential distribution [7], a new Lindley-Exponential distribution [8], the Lomax-exponential distribution [9], the transmuted odd generalized exponential-exponential distribution [10], the transmuted exponential distribution [11], transmuted inverse exponential distribution[12], the odd generalized exponential-exponential distribution [13], the transmuted Weibull-exponential distribution [14] and the Weibull-exponential distribution [15]. Following these recent publications and considering our desire to improve the flexibility of the Exponential Inverse Exponential distribution which was found to be an improvement over the Inverse Exponential distribution, this article proposes a new distribution called the exponentiated exponential inverse exponential distribution.

The probability density function (pdf) of the Exponential Inverse Exponential distribution (ExInExD) according to Oguntunde PE et al. [6] is defined by

$$g(x)=\frac{\alpha\theta}{x^2}\frac{e^{-\frac{\theta}{x}}}{\left[1-e^{-\frac{\theta}{x}}\right]^2}e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)} \tag{1}$$

The corresponding cumulative distribution function (cdf) of Exponential Inverse Exponential distribution (ExInExD) is given by

$$G(x)=1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)} \tag{2}$$

where, $x>0,\theta>0,\alpha>0$; α is the shape parameter and θ is a scale parameter.

This distribution was found to be better than the exponential and inverse exponential distribution. A study of its important mathematical and statistical properties as well as the maximum likelihood estimation of its parameters and its applications using real life datasets can be found in [6].

The aim of this paper is to introduce a new continuous distribution called the Exponentiated Exponential Inverse Exponential distribution (ExpExInExD) using the proposed method [16]. This paper is organized in different sections as follows: definition of the new distribution with its plots is provided in section 2. Section 3 derived some properties of the proposed distribution. Section 4 looks at distribution of order statistics. The estimation of parameters using maximum likelihood estimation (MLE) is presented in section 5. An application of the new model with other existing distributions to mother-to-child HIV transmission rate data is done in section 6 and a conclusion is given in section 7.

2. THE EXPONENTIATED EXPONENTIAL INVERSE EXPONENTIAL DISTRIBUTION (ExpExInExD)

According to Mudholkar GS et al. [16], a random variable X is said to have an exponentiated form of distribution function if its cdf and pdf are respectively given by;

$$F(x) = \left[G(x)\right]^{\beta} \tag{3}$$

and

$$f(x) = \beta g(x)\left[G(x)\right]^{\beta-1} \tag{4}$$

where; $x > 0$, and β is the extra shape parameter, $G(x)$ and $g(x)$ are the cdf and pdf of any continuous distribution to be modified respectively.

Putting equation (1) and (2) into equation (3) and (4) and simplifying, we obtain the cdf and pdf of the ExpExInExD given in equation (5) and (6) respectively as follows:

$$F(x) = \left[1 - \mathrm{e}^{-\alpha\left(\frac{\mathrm{e}^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta} \tag{5}$$

and

$$f(x) = \frac{\alpha\theta\beta}{x^2}\frac{\mathrm{e}^{-\frac{\theta}{x}}}{\left[1-e^{-\frac{\theta}{x}}\right]^2} e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\left[1 - e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta-1} \tag{6}$$

where $x > 0, \alpha > 0, \theta > 0, \beta > 0$, α and β are the shape parameters and θ is the scale parameter.

Plots of the pdf and cdf of the ExpExInExD using some parameter values are presented in Fig. 1 as follows.

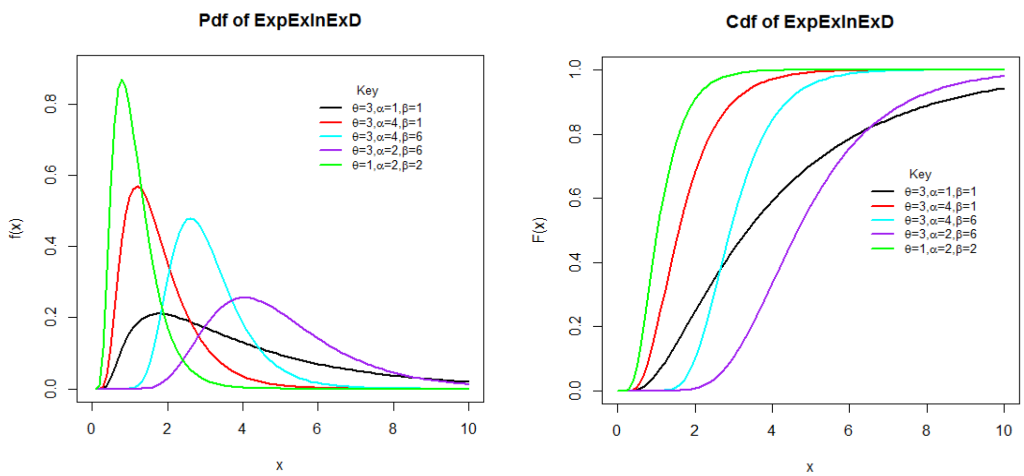

Fig. 1. PDF and CDF of the ExpExInExD for different values of the parameters

From the figure above, it can be seen that the pdf of ExpExInExD is positively skewed and takes various shapes depending on the parameter values. Also, from the above plot of the cdf, it is clear that the cdf equals to one when *X* approaches infinity and equals zero when *x* tends to zero as normally expected.

3. SOME PROPERTIES OF EXPEXINEXD

In this section, some properties of the ExpExInExD are derived and discussed as follows:

3.1Quantile Function

According to Hyndman RJ et al. [17], the quantile function for any distribution is defined in the form $Q(u) = X_q = F^{-1}(u)$ where $Q(u)$ is the quantile function of *F*(*x*) for $0 < u < 1$.

Taking *F(x)* to be the cdf of the ExpExInExD and inverting it as above will give us the quantile function as follows:

$$F(x) = \left[1 - e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta} = u \qquad (7)$$

Simplifying equation (7) above and solving for X presents the quantile function of the ExpExInExD as:

$$Q(u) = \theta\left\{\log\left[\left(-\frac{\log\left(1-u^{\beta}\right)}{\alpha}\right)^{-1} + 1\right]\right\}^{-1} \qquad (8)$$

This function is used for obtaining some moments like skewness and kurtosis as well as the median and for generation of random variables from the distribution in question.

3.2 Skewness and Kurtosis

This paper presents the quantile based measures of skewness and kurtosis due to non-existence of the classical measures in some cases.

According to Kenney JF et al. [18], the Bowley's measure of skewness based on quartiles is given by:

$$SK = \frac{Q\left(\frac{3}{4}\right) - 2Q\left(\frac{1}{2}\right) + Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right) - Q\left(\frac{1}{4}\right)} \tag{9}$$

Also, the Moors kurtosis based on octiles proposed by Moors JJ [19] and is given by;

$$KT = \frac{Q\left(\frac{7}{8}\right) - Q\left(\frac{5}{8}\right) - Q\left(\frac{3}{8}\right) + \left(\frac{1}{8}\right)}{Q\left(\frac{6}{8}\right) - Q\left(\frac{1}{8}\right)} \tag{10}$$

Where $Q(.)$ is obtainable with the help of equation (8).

3.3 Reliability Analysis of the ExpExInExD

Under this section, a derivation and study of the survival (or reliability) function and the hazard (or failure) rate function are presented.

The Survival function describes the likelihood that a system or an individual will not fail after a given time. Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \tag{11}$$

Applying the cdf of the ExpExInExD in (11), the survival function for the ExpExInExD is obtained as:

$$S(x) = \left\{ 1 - \left[1 - e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)} \right]^{\beta} \right\} \tag{12}$$

The plot for the survival function of the ExpExInExD using different parameter values is shown in Fig. 2 below:

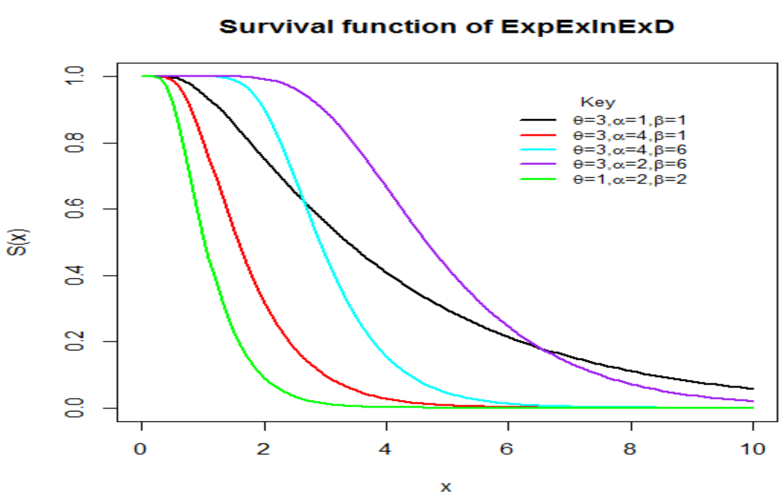

Fig 2. Survival function of the ExpExInExD at different parameter values

The plot in Fig. 2 shows that the probability of survival is always sure at initial time or early age and it decreases as time increases up to zero (0) at infinity.

Hazard function is the probability that a component will fail or die for an interval of time. The hazard function is defined as;

$$h(x)=\frac{f(x)}{S(x)}=\frac{f(x)}{1-F(x)} \tag{13}$$

Meanwhile, the expression for the hazard rate of the ExpExInExD is given by:

$$h(x)=\frac{\alpha\theta\beta e^{-\frac{\theta}{x}}e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta-1}}{x^2\left[1-e^{-\frac{\theta}{x}}\right]^2\left\{1-\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta}\right\}} \tag{14}$$

where $\alpha,\theta,\beta>0$.

A plot of the hazard function for arbitrary parameter values is presented in Fig. 3 as follows

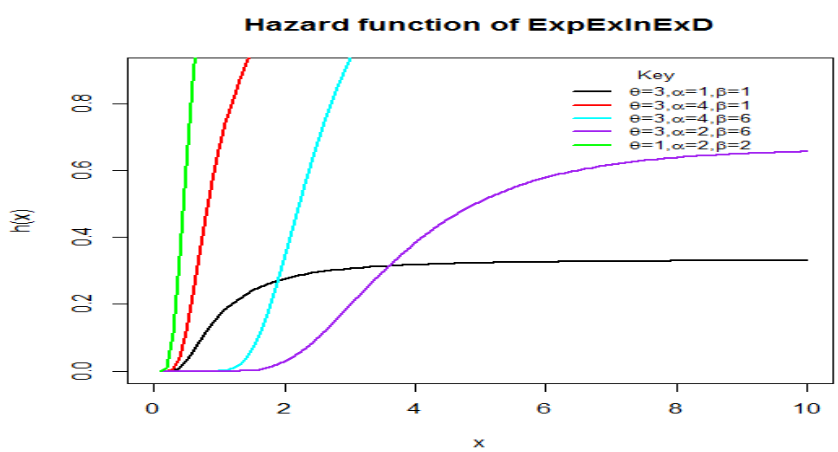

Fig. 3. Hazard function of the ExpExInExD

The figure above revealed that the ExpExInExD has increasing as well as constant failure rate which implies that the probability of failure for any random variable following a ExpExInExD increases as time increases, that is, probability of failure or death increases with age. It also shows that the failure rate could be constant after sometimes depending on the parameter values.

4. DISTRIBUTION OF ORDER STATISTICS

Suppose $X_1, X_2, \ldots.., X_n$ is a random sample from the ExpExInExD and let $X_{1:n}, X_{2:n}, \ldots.., X_{i:n}$ denote the corresponding order statistic obtained from this same sample. The *pdf*, $f_{i:n}(x)$ of the i^{th} order statistic can be obtained by

$$f_{i:n}(x)=\frac{n!}{(i-1)!(n-i)!}\sum_{k=0}^{n-i}(-1)^k\binom{n-i}{k}f(x)F(x)^{k+i-1} \tag{15}$$

Using (5) and (6), the pdf of the i^{th} order statistic $X_{i:n}$, can be expressed from (15) as;

$$f_{i:n}(x)=\frac{n!}{(i-1)!(n-i)!}\sum_{k=0}^{n-i}(-1)^k\binom{n-i}{k}\left[\frac{\alpha\theta\beta e^{-\frac{\theta}{x}}e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}}{x^2\left[1-e^{-\frac{\theta}{x}}\right]^2}\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta-1}\right]\left[\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta}\right]^{i+k-1} \tag{16}$$

Hence, the pdf of the minimum order statistic $X_{(1)}$ and maximum order statistic $X_{(n)}$ of the ExpExInExD are respectively given by:

$$f_{1:n}(x)=\frac{n!}{(n-1)!}\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\left[\frac{\alpha\theta\beta e^{-\frac{\theta}{x}}e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}}{x^2\left[1-e^{-\frac{\theta}{x}}\right]^2}\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta-1}\right]\left[\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta}\right]^{k} \tag{17}$$

and

$$f_{n:n}(x)=n\left[\frac{\alpha\theta\beta e^{-\frac{\theta}{x}}e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}}{x^2\left[1-e^{-\frac{\theta}{x}}\right]^2}\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta-1}\right]\left[\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x}}}{1-e^{-\frac{\theta}{x}}}\right)}\right]^{\beta}\right]^{n-1} \tag{18}$$

5. ESTIMATION OF UNKNOWN PARAMETERS OF THE EXPEXINEXD

In this section, the estimation of the parameters of the ExpExInExD has been done by using the method of maximum likelihood estimation (MLE). Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the ExpExInExD with unknown parameters α,θ and β defined previously.

The likelihood function of the ExpExInExD using the pdf in equation (6) is given by;

$$L\left(\underline{X}/\alpha,\theta,\beta\right)=\left(\alpha\theta\beta\right)^n\prod_{i=1}^{n}\left(x_i^{-2}\left[1-e^{-\frac{\theta}{x_i}}\right]^{-2}e^{-\frac{\theta}{x_i}}e^{-\alpha\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)}\right)\prod_{i=1}^{n}\left[\left(1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)}\right)^{\beta-1}\right] \tag{19}$$

Let the natural logarithm of the likelihood function be, $l=\log L\left(\underline{X}\mid\alpha,\theta,\beta\right)$, therefore, taking the natural logarithm of the function above gives:

$$l=n\log\alpha+n\log\theta+n\log\beta-2\sum_{i=1}^{n}\log x_i-2\sum_{i=1}^{n}\log\left(1-e^{-\frac{\theta}{x_i}}\right)-\theta\sum_{i=1}^{n}x^{-1}{}_i-\alpha\sum_{i=1}^{n}\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)+(\beta-1)\sum_{i=1}^{n}\log\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)}\right] \tag{20}$$

Differentiating l partially with respect to α,θ and β respectively gives the following results;

$$\frac{\partial l}{\partial\alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)+\left(\beta-1\right)\sum_{i=1}^{n}\left\{\frac{\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)e^{-\alpha\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)}}{1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)}}\right\} \tag{21}$$

$$\frac{\partial l}{\partial \theta}=\frac{n}{\theta}-\sum_{i=1}^{n}x_i^{-1}+2\sum_{i=1}^{n}\left\{\frac{e^{-\frac{\theta}{x_i}}}{x_i\left(1-e^{-\frac{\theta}{x_i}}\right)}\right\}+\alpha\sum_{i=1}^{n}\left\{\frac{e^{-\frac{\theta}{x_i}}}{x_i\left[1-e^{-\frac{\theta}{x_i}}\right]^2}\right\}+\alpha(\beta-1)\sum_{i=1}^{n}\left\{\frac{e^{-\frac{\theta}{x_i}}}{x_i\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)}\right]\left[1-e^{-\frac{\theta}{x_i}}\right]^2}\right\} \quad (22)$$

$$\frac{\partial l}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\log\left[1-e^{-\alpha\left(\frac{e^{-\frac{\theta}{x_i}}}{1-e^{-\frac{\theta}{x_i}}}\right)}\right] \quad (23)$$

Making equation (21), (22) and (23) equal to zero (0) and solving for the solution of the non-linear system of equations produce the maximum likelihood estimates of parameters α, θ and β. However, these solutions cannot be obtained manually except numerically with the aid of suitable statistical software like *R*.

6. APPLICATION TO MOTHER-TO-CHILD HIV TRANSMISSION RATE (MTCHIVTR)

This section presents a dataset on the rate of mother-to-child transmission of HIV (Human Immunodeficiency Virus) in Nigeria from the year 2000 to the year 2019. The descriptive statistics and graphical summary of the dataset is also presented.

The mother-to-child HIV transmission rate per 1,000 of population in Nigeria between 2000 and 2019 is as given below.

37.35, 37.08, 37.00, 36.98, 36.79, 36.75, 34.35, 32.96, 31.84, 30.35, 30.53, 28.96, 26.71, 22.50, 19.84, 20.04, 19.44, 20.82, 22.09, 22.16

Data source: www.data.unicef.org

The following table and figures present a critical exploration of the above dataset with some important discussions:

Following the summary of the descriptive statistics in Table 1 and the histogram, box plot, density and normal Q-Q plot generally referred to as graphical summary in Fig. 4, it is seen that the rate of transmission of HIV from mother to child is bimodal and approximately normally distributed.

The following figure shows the trend in the rate of mother-to-child HIV transmission from 2000 to 2019 using a bar chart.

After checking the distribution of the dataset in Fig. 4, the bar chart in Fig. 5 reveals the trend in the rate of mother-to-child transmission of HIV which indicates that mother-to-child HIV transmission was a very big problem from the year 2000 to 2005 with a non-decreasing rate. Meanwhile, there came a slightly decreasing trend in the rate of HIV transmission from mother to child as from the year 2006 to 2014, however, what we have from the year 2015 to 2019 is certainly an increasing pattern in the rate of mother-to-child transmission of HIV which suggests that more efforts need to be put in place to adequately reduce or eradicate the increasing rate of mother-to-child HIV transmission in Nigeria.

Considering the increasing rate of mother-to-child HIV transmission and the flexibility of the proposed distribution, this study fits the Exponentiated Exponential Inverse Exponential distribution (ExpExInExD) to the above dataset in comparison with other existing probability distributions such as Exponential Inverse Exponential distribution (ExInExD), Odd Lindley Inverse Exponential distribution (OLINExD), Lindley distribution (LIND), Inverse Exponential distribution (InExD) and Exponential distribution (ExD).

To identify the most efficient or most fitted distribution to the MTCHIVTR dataset, the following model selection criteriawere used which include the value of the log-likelihood function evaluated at the MLEs (ℓ), Akaike Information Criterion, *AIC,* Consistent Akaike Information Criterion, *CAIC*, Bayesian Information Criterion, *BIC*, Hannan Quin Information Criterion, *HQIC*, Anderson-Darling (A*), Cramèr-Von Mises (W*) and Kolmogorov-smirnov (K-S) statistics. More about the statistics A*,W* and K-Scan be seen in [20]. Some of these statistics are computed using the following formulas:

Table 1. Descriptive statistics for the dataset

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Dataset A	20	19.44	22.14	30.44	36.76	29.23	37.35	47.55	-0.18919	-1.55278

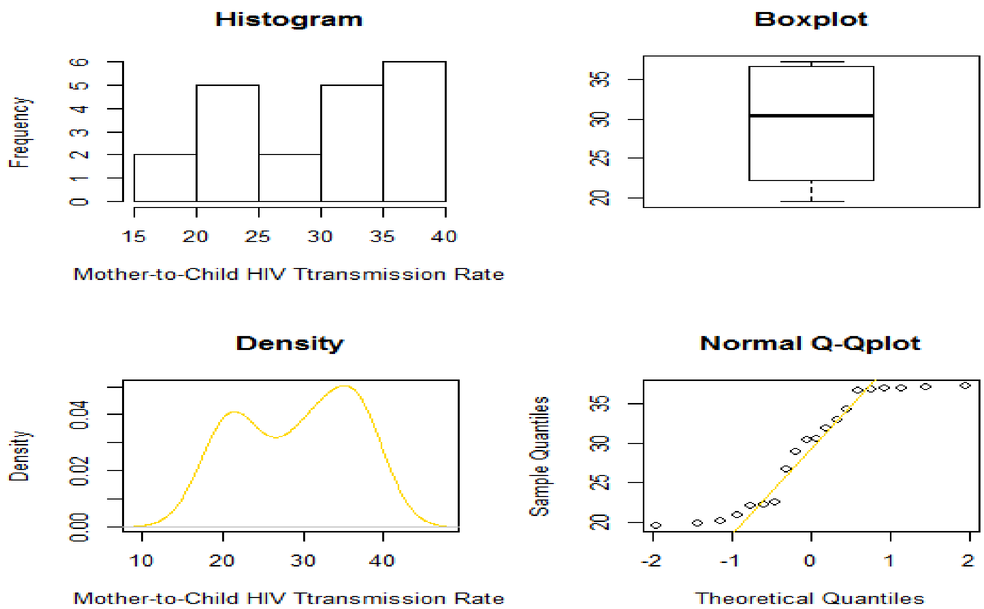

Fig. 4. A graphical summary of the dataset

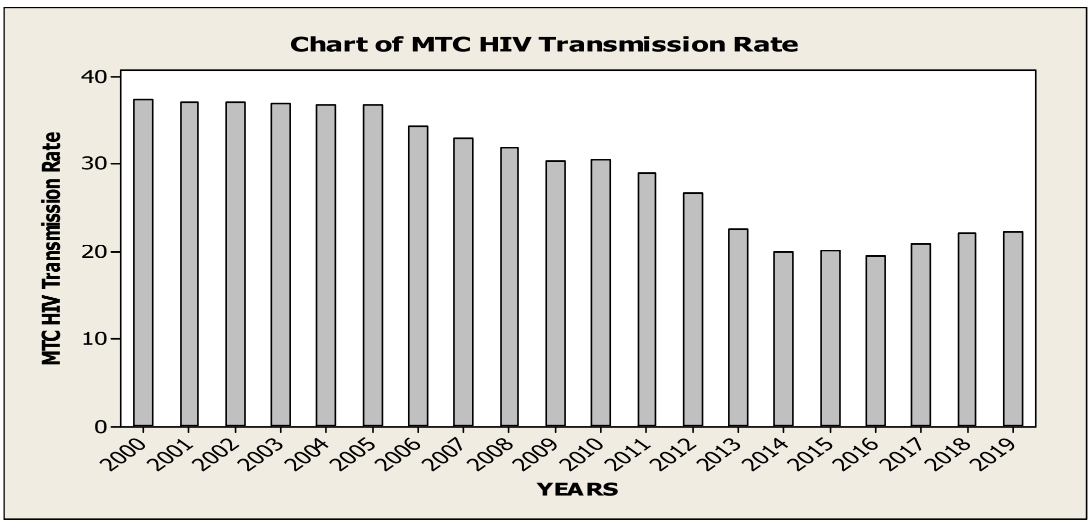

Fig. 5. A Bar chart showing the Trend of Mother-to-child HIV transmission rate in Nigeria from 2000 to 2019

$AIC = -2\ell + 2k$, $BIC = -2\ell + k\log(n)$, $CAIC = -2\ell + \frac{2kn}{(n-k-1)}$ and $HQIC = -2\ell + 2k\log\left[\log(n)\right]$

Where ℓ denotes the value of log-likelihood function evaluated at the *MLEs*, *k* is the number of model parameters and *n* is the sample size. To cut the long story short, the distribution with the lowest values of these criteria is considered to be the best model that fit the dataset. Also, all the required computations are performed using the R package "Adequacy Model" which is freely available from http://cran.r-project.org/web/packages/AdequacyModel/AdequacyModel.pdf. The results from this R package and the commands are shown in tables as follows:

Tables 2 lists the Maximum Likelihood Estimates of the model parameters, Table 3 presents the statistics AIC, CAIC, BIC and HQIC while A*,W* and K-S for the fitted models are given in Table 4.

Table 2. Maximum likelihood parameter estimates for the dataset

Distribution	$\hat{\theta}$	$\hat{\alpha}$	$\hat{\beta}$
ExpExInExD	9.583134	1.080386	9.188880
LIND	-	0.0663024	-
OLINExD	3.0133921	0.2039191	-
ExD	0.03448022	-	-
ExInExD	9.1300972	0.4041777	-
InExD	6.310545	-	-

Table 3. The statistics ℓ, AIC, CAIC, BIC and HQIC based on the dataset used

Distribution	$\hat{\ell}$	AIC	CAIC	BIC	HQIC	Ranks
ExpExInExD	69.41392	144.8278	146.3278	147.815	145.411	1st
LIND	80.93501	163.87	164.0922	164.8657	164.0644	2nd
OLINExD	80.99396	165.9879	166.6938	167.9794	166.3767	3rd
ExD	87.50246	177.0049	177.2271	178.0006	177.1993	4th
ExInExD	84.601	173.202	173.9079	175.1935	173.5908	5th
InExD	101.6024	205.2049	205.4271	206.2006	205.3992	6th

Table 4. The A^*, W^*,K-S statistic and P-values based on the dataset used

Distribution	A^*	W^*	K-S	P-Value (K-S)	Ranks
ExpExInExD	1.095666	0.1673933	0.20667	0.3152	1st
LIND	1.042629	0.1535517	0.39133	0.002858	2nd
OLINExD	1.04269	0.1535964	0.40427	0.001824	3rd
ExD	1.044671	0.1541544	0.48844	6.382e-05	4th
ExInExD	1.053079	0.1566282	0.49046	5.832e-05	5th
InExD	1.115619	0.1725355	0.7228	3.89e-11	6th

The following figure presents a histogram and estimated densities and cdfs of the fitted models to the dataset.

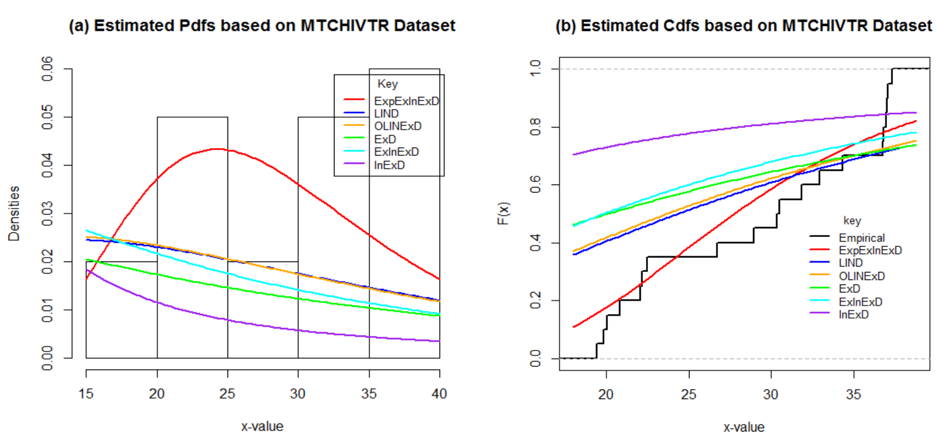

Fig. 6. Histogram and plots of the estimated densities and cdfs of the fitted distributions to the dataset

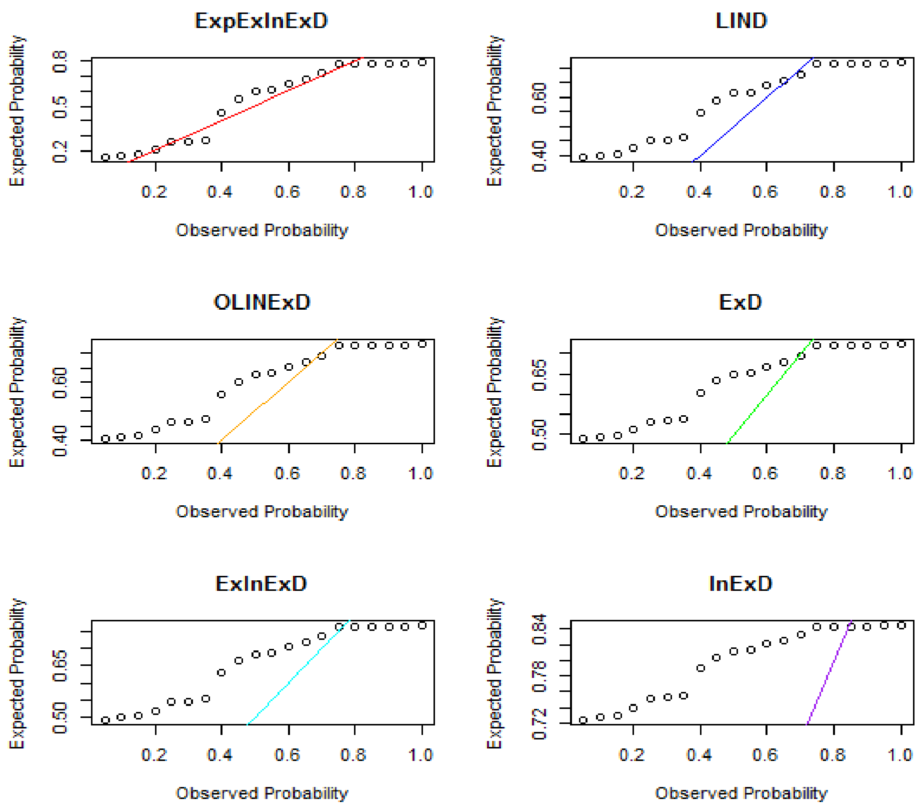

Fig. 7. Probability plots for the six fitted distributions based on the MTCHIVTR dataset

It is observed from the results in Table 2 that the proposed distribution (ExpExInExD) performs better than the other fitted distributions based on the values of the first four information criteria (AIC, CAIC, BIC and HQIC). Also, deciding on the best distribution based on the statistics in Tables 4, we conclude again that the ExpExInExD has the minimum values of A^*, W^* and K-S statistic compared to every other model fitted to the dataset. From all these model selection criteria, it is clear that the ExpExInExD has the overall best fit to the mother-to-child HIV transmission rate dataset and therefore it is chosen as the most adequate model for explaining this dataset as considered in this study.

The histogram of the dataset together with the fitted densities and estimated cumulative distribution functions given in Fig. 6 also confirm that the proposed model analyses the dataset better than the LIND, OLINExD, ExInExD, ExD and the conventional InExD. Also, the probability plots presented in Fig. 7 provide evidences that the proposed distribution (ExpExInExD) is more flexible than the other five distributions (LIND, OLINExD, ExInExD, ExD and InExD) as already revealed previously in Tables 3 and 4 as well as Fig. 6 respectively.

These results above also prove the fact that adding parameter(s) to most continuous probability distributions leads to increase in its flexibility in modeling real life data as it has already been reported by many other authors in the previous studies.

7. CONCLUSION

A new extension of the inverse exponential distribution known as “Exponentiated Exponential Inverse Exponential distribution” has been proposed in this paper. Some important properties of the proposed distribution have been investigated. These properties include quantile function, coefficient of skewness and kurtosis, survival function and hazard function. The paper also obtained the distribution of minimum and maximum order statistics based on the proposed distribution. It also estimated the unknown parameters of the model by method of maximum likelihood estimation. The proposed distribution has been applied to a dataset on mother-to-child HIV transmission rate in Nigeria from the year 2000 to 2019 in comparison with other existing distributions. A brief exploratory analysis of the dataset shows that there is an increasing trend in the rate of mother-to-child transmission of HIV in Nigeria and we demand for immediate actions from relevant health agencies. The results from the fitted models based on the MTCHIVTR dataset reveal that the exponentiated exponential inverse exponential distribution fits the dataset much better than the other five fitted distributions. This also indicates that the proposed model could be used for analysis of HIV surviving patients. Therefore it is very clear that the new model is more flexible than the other five models considered in this study and should be used for modeling other real life situations most especially in medical sciences.

CONSENT

It is not applicable.

ETHICAL APPROVAL

It is not applicable.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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