Journal of Engineering Research and Reports

20(1): 107-125, 2021; Article no.JERR.64463
ISSN: 2582-2926

Optimizing the Tensile Strength of Polystyrene Lightweight Concrete: A Panacea for Sustainable Housing Development in Developing Countries

S. E. Ubi[1*]

[1]*Department of Civil Engineering, Faculty of Engineering, Cross River University of Technology, Calabar, Nigeria.*

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/JERR/2021/v20i117254
Editor(s):
(1) Dr. Tian- Quan Yun, South China University of Technology, China.
Reviewers:
(1) Anil Kumar, Lovely Professional University, India.
(2) C. Sashidhar, JNTUA College of Engineering, India.
Complete Peer review History: http://www.sdiarticle4.com/review-history/64463

Original Research Article

Received 02 November 2020
Accepted 08 January 2021
Published 29 January 2021

ABSTRACT

The use of polystyrene beads in concrete applications has been limited due to its perceived low strength properties. Tensile strength test is an important test that determines the vulnerability of concrete to tensile cracking due to the weight of the structural load. Water, sand, coarse aggregates, expanded polystyrene beads, and ordinary Portland cement are the materials used for this study. All the materials were batched according to their weight, except for polystyrene and coarse aggregates which were batched in volume after mixing them together. The polystyrene partial replacement level was considered at 12% of the coarse aggregate volume. The model equation adopted for this study was based on Scheffe's {4, 2} simplex lattice design for both Pseudo component and component proportional models. The actual model was developed from the 28th day test result. The Mathlab and Minitab 16 software were used in this study to generate the actual mix ratios. The results obtained showed that both Pseudo component and component proportional models both produced an average split tensile strength of about 5.10N/mm^2. This implied that the results of this study produced a split tensile strength result that varied between 18% - 19% of its compressive strength result. This showed that the materials and the mix ratios optimized in this study are suitable as building blocks for residential low rising buildings and as partition slaps for high rising buildings. The lightweight property makes it highly suitable for large scale application in high rising structures as internal partition slaps only.

**Corresponding author: Email: stanleyubi@crutech.edu.ng;*

Keywords: Expanded polystyrene; Scheffe's models; lightweight concrete; optimisation; tensile strength.

1. INTRODUCTION

Natural resources are fixed and finite, but human populations are on the increase on a daily basis. As populations are growing, there is the corresponding rising demand for basic needs such as housing. In 1943, Abraham Maslow in his theory of human motivation, identified five precursors to human motivation, with each level serving as a precursor in themselves to an individual's motivation towards achieving the next level. Ranked in a pyramid hierarchical order, at the base of the pyramid were the physiological needs, while in an ascending order were safety needs, love and belonging, esteem and self-actualization. The hierarchy depicted the relevancy of each level from the base. In order words, the most basic level is the base. These included basic human needs such as food, water, clothing, shelter, sleep etc. While this theory might have come under several criticism, it invariably underscores the point that as human populations are growing, the need for housing is also growing, while the construction materials are finite. Hence, there is the need to develop alternate means that will guarantee the sustainability of these finite resources amidst the environmental concerns of their exploitation. Consequently, the need to identify reusable synthetic waste materials with concrete bonding abilities as substitute to natural aggregates. One of such synthetic aggregates is the expanded polystyrene beads [1]. Several researchers like (Kharun & Svintsov, 2017; [2]; [3]; Chen & Fang, 2011; [4]; Sabaa & Ravindrarajah, 1997), have all explored various behaviors of polystyrene concrete under different mechanical and environmental conditions, but it remains to be seen how all of these properties can be mathematically modelled.

Expanded polystyrene (EPS) beads are non-biodegradable cellular lightweight plastic materials. They consist of well-arranged spherically shaped particles that are made up of about 98% polystyrene and 2% air. Due to their closed cell structure, they do not easily or completely absorb water. Their unique characteristics makes them principally suitable for impact resistance as well as sound and thermal insulation. This is why Polystyrene foam commonly serve as packaging materials for fragile items in the packaging industry. Since they are commonly used for packaging household items such as electronics, they are commonly available. Given that they are non-biodegradable, their availability is further compounding the existing problem of plastic waste generation and disposal ([5]; [6]; [7] and [8].

The use of polystyrene beads as concrete aggregate has been generally limited due to its perceived low strength properties [4]. Obtaining the right mix ratio that will yield the required strength properties vis-à-vis the tensile strength require several trial mixes. This makes the process highly painstaking, time consuming and cost intensive. However, the studies of Ubi, Okafor and Mama [9] have debunked this long standing assumption by using mathematical optimization models to accurately predict the compressive strength of polystyrene lightweight concrete at the least cost and time. This therefore implies that polystyrene beads stand a chance of producing acceptable concrete that is suitable for wide scale structural applications [10].

According to the Ubi et al., [9] successfully optimized the compressive strength of polystyrene concrete to suite all forms of residential applications. Taking it a step further to optimize the corresponding tensile strength will invariably prove that concrete produced from waste materials such as polystyrene can withstand tensile cracking as well as environmental adversity, while maintaining its lightweight property. This will contribute to solving the global challenge of housing needs. According to the UN habitat [11] report, over 1.6 billion persons globally are in dire need of adequate housing facility. Many of such are attributable to the high cost of housing which is consequent upon the high cost of materials. Since getting the right mix ratios to produce optimized strength properties may no longer be a problem, it is safe to hypothesize that concrete from waste materials vis-à-vis polystyrene materials will help in cushioning this effect. Kumar et al. [12] reported on the optical study of polyaniline/ polystyrene composite films. Polyaniline-carbon Nanotube composites: Preparation methods, properties and applications was also examined by Kumar et al., [13].

Most housing policies in developing countries center more on the tenure structure and in some instances on the affordability, while very little attention is paid to identifying the need to develop alternative materials that can guarantee sustainable development. Obviously as human populations continue grow arithmetically alongside polystyrene waste generation, there is the need to turn such waste into more useful applications. Government efforts in terms of policy formulations is required in this direction.

2. MIXTURE MODELS AND EXPERIMENTS

When an experiment is dependent on the proportions of its constituents' materials rather than on the volume or mass of its constituent materials, such an experiment is said to be a mixture experiment [14]. As a general rule for such experiments, the sum of proportion for all constituents' materials must equal unit (1). Secondly, none of its constituents is expected to have a negative value. This statement can be expressed mathematically as follows;

$$X_1 + X_2 + \cdots + X_q = \sum_{I=1}^{q} X_i = 1 \qquad \text{(i)}$$

$$0 \leq X_i \leq 1 \qquad \text{(ii)}$$

Where

q is the number of mixture components.

X_i *(i = 1 to q)* is the mass or volume of the proportion of the i^{th} constituent in the experiment. However, since the totality of the component proportions bust add up to 1, only *q-1* of the variables or constituents can be independently chosen. From Equation (iii),

$$X_q = 1 - \sum_{i=1}^{q-1} X_i \qquad \text{(iii)}$$

Assuming the responses – which in this instance is the 28th day tensile strength – is denoted by *y* and $X_1, X_2, X_3, X_4 \ldots X_q$ are the mix components of the experiment, then the equation can be rewritten as:

$$y = F\left(X_1, X_2, X_3 \ldots\ldots\ldots.., X_q\right) \qquad \text{(iv)}$$

Mixture models have been applied in many real life applications to solve problems in such areas as in pharmacy, food industry, agriculture and engineering. This could be found in the works of Piepel and Redgate [15] who applied it in the determination of oxide composition in cement clinker.

2.1 Scheffe's Simplex Lattice Design

According to Goelz [16] and Ubi et al., [17,18], a simplex is a geometric figure with the number of vertices being one more than the number of variable factor space, *q*. It is a projection of n-dimensional space onto an n-1 dimensional coordinate system. This implies that if *q* = 1, then the number of vertices is 2 and the simplex in this case is a straight line. If *q* = 2, then the number of vertices is 3 and the simplex will become a triangle. The simplex becomes a tetrahedron when *q* = 3 and above. A lattice is an ordered arrangement of points in a regular pattern. Scheffe [19], however, expanded and generalized the simple lattice design. His work has been often regarded as the pioneering work in simplex lattice mixture design. Lattice designs are presently often referred to as Scheffe's simplex lattice designs. He assumed that each components of the mixture resides on a vertex of a regular simplex-lattice with *q*-1 factor space. If the degree of the polynomial to be fitted to the design is *n* and the number of components is *q* then the simplex lattice, also called a *{q, n}* simplex will consist of uniformly spaced points whose coordinates are defined by the following combinations of the components: the proportions assumed by each component take the *n*+1 equally spaced values from 0 to 1, that is;

$$X_i = 0, \quad \frac{1}{n}, \quad \frac{2}{n}, \ldots\ldots\ldots..1 \qquad \text{(v)}$$

The simplex lattice consists of all possible combinations of the components where the proportions of Equation *(vi)* for each component are used [14]. The second degree Scheffe's polynomial for *q* components is given as:

$$y = \sum_{1\leq i\leq q} \beta_i X_i \sum_{1\leq i\leq j\leq q} \beta_{ij} X_i X_j \qquad \text{(vi)}$$

The number of terms in the Scheffe's polynomial, *N* is the minimum number of experimental runs necessary to determine the polynomial coefficients and is given as:

$$N = C_n^{(q+n-1)} = \frac{(q+n-1)!}{(q-1)!(n)!} \qquad \text{(vii)}$$

Consider a four component mixture. The factor space is a tetrahedron. If a second degree polynomial is to be used to define the response over the factor space then each component (X_1,

$X_2 \ldots X_4$) must assume the proportions $X_i = 0, \ 1/2$, and 1. The {4, 2} simplex-lattice consists of the ten points at the boundaries and the vertices of the tetrahedron: (X_1, X_2, X_3, X_4) = (1,0,0,0) , (0,1,0,0) , (0,0,1,0), (0,0,0,1), (1/2,1/2,0,0), (1/2,0,1/2,0), (1/2,0,0,1/2), (0,1/2,1/2,0), (0,1/2,0,1/2) and (0,0,1/2,1/2). The four points defined by (1,0,0,0) , (0,1.0,0) , (0,0,1,0) and (0,0,0,1), represent single component mixtures at the vertices of the tetrahedron.(1,0,0, 0). Therefore the governing regression equation of Scheffe for predicting tensile strength from the 4 mixture components is as follows:

$$y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \quad \text{(viii)}$$

3. MATERIALS AND METHODS

Materials for this study included; Water, Ordinary Portland cement, coarse aggregate (granite chippings), fine aggregates (river sand) and Expanded Polystyrene beads. Cement was obtained from a major Lafarge brand cement dealer in Calabar. Potable water conforming to the specification of EN 1008: [20] was used for all specimen mixture and curing. Sand was obtained from the Calabar River beach in Calabar, Nigeria. Coarse Aggregate was obtained from the quarry site of Crush Rock Industries at Akamkpa, Cross River State, Nigeria. Lastly, the polystyrene beads were obtained from a local distributor in Owerri, Nigeria. The materials were batched according to their weights, except for coarse aggregates and polystyrene beads which were mixed and batched together as a single material in volume. Hence, the total number of components was 4 and a second degree polynomial was used in designing the experiments. That is, q = 4 and n = 2 and the first 20 mix ratios were utilized. The constituents were manually mixed in the laboratory and the results used for model optimization were based on the 28th day test. All specimens were cured based on NIS 87 [21].The experiment was conducted in Strength of Material Lab, Workshop five (5) Cross River University of Technology Calabar, Nigeria. Cylindrical beams of size 300x150x150mm were prepared for split tensile strength in accordance with BS 1881: Part 117 [22] and IS: 5816-1970 [23]. Equation *(ix)* was used for computing the lateral tensile strength of concrete specimen.

$$f_t = \frac{2F}{\pi L d} \quad \text{(ix)}$$

Where,
F is the maximum breaking load, L the length of the test specimen and d is the diameter of the test specimen. The Scheffe's regression model as indicated in equation *(Viii)* was used to model the regression coefficients of the analysis, while the regression analysis was done using SPSS computer software. The variable were transformed into actual ratios according to Ubi et al., [9]. A computer program known as the optimizer was designed using Matlab 2010 to automate the process of obtaining the optimization results.

4. RESULTS AND DISCUSSION

4.1 Scheffe's Pseudo Component Model

Table 1 and Table 2 show the estimated regression coefficients with the associated statistics and the Anova table respectively. Table 3 shows the observed strengths and the fitted values (predicted) along with the residuals.

4.1.1 Model equation

It is seen in Table 1 that both the linear and quadratic regression sources are significant at 95% confidence limit since each has a *p-value* less than 0.05. The quadratic model is chosen since it is of a higher degree than the linear model. Thus the coefficients of the Scheffe's second degree polynomial are given as:

$$\beta_1 = 2.470, \quad \beta_2 = 1.980, \quad \beta_3 = 1.608, \quad \beta_4 = 1.316, \quad \beta_{12} = -0.656, \quad \beta_{13} = -0.844, \quad \beta_{14} = -0.391, \quad \beta_{23} = -0.415, \quad \beta_{24} = 0.416, \quad \beta_{34} = 0.305$$

If we let the components water, cement, sand and polystyrene and granite chipping aggregates to be represented respectively by X_1, X_2, X_3 and X_4, then the model equation in terms of pseudo units is:

$$Y = 2.470X_1 + 1.980X_2 + 1.608X_3 + 1.316X_4 - 0.656X_1X_2 - 0.844X_1X_3 - 0.391X_1X_4 - 0.415X_2X_3 + 0.416X_2X_4 + 0.305X_3X_4 \quad \text{(x)}$$

Table 1. Estimated regression coefficients for compressive strength

Model		Unstandardized coefficients		Standardized coefficients	T	Sig.
		B	Std. error	Beta		
1	X1	2.470	.189	.513	13.098	.000
	X2	1.980	.188	.372	10.553	.000
	X3	1.608	.180	.344	8.951	.000
	X4	1.316	.180	.291	7.302	.000
	X1 * X2	-.656	.799	-.027	-.821	.431
	X1 * X3	-.844	.763	-.036	-1.106	.294
	X1 * X4	-.391	.757	-.017	-.516	.617
	X2 * X3	-.415	.814	-.018	-.510	.621
	X2 * X4	.416	.795	.019	.524	.612
	X3 * X4	.305	.799	.012	.382	.711

(Scheffe's pseudo components model)

Table 2. Analysis of variance for compressive strength

Model		Sum of squares	df	Mean square	F	Sig.
1	Regression	62.415	10	6.241	169.901	.000^c
	Residual	.367	10	.037		
	Total	62.782^d	20			

(Scheffe's pseudo component model)

4.1.2 Test for lack-of-fit

Table 2 shows that there is insignificant lack-of-fit, the *p-value* for lack-of-fit being 0.00 which is less than 0.05. The conclusion, therefore, is that Equation (*x*) is adequate for predicting the 28th day strength of expanded polystyrene concrete. The other statistics in Table 1, lend credence to the adequacy of the model.

4.1.3 Experiment versus model tensile strength (pseudo component model)

Table 4 shows the twenty mix ratios as well as their corresponding laboratory and modelled tensile strength respectively for the pseudo component model. The model result showed that the tensile strength for the three mixes were 1.90MPa, 1.96MPa and 1.68 MPa respectively. As per ASTM C 496, the compressive strength of residential and commercial structures is between 17 – 28MPa. Also, as per ASTM C 496, [24] minimum tensile strength is 10 per cent of the compressive strength, this puts the tensile strength of residential and commercial structures between 1.7 – 2.8MPa. While this study addresses the tensile strength, it can be seen that the three mixes at mix 12, mix 18 and mix 20 produces the acceptable tensile strength for residential structures.

4.1.4 Optimization result for pseudo component model

Data in Table 5 shows the optimized mixed ratios generated by the optimizer program using the data from Apendix1a & b, and Table 4. The mix ratios yielded an average optimized split tensile strength of about 5.1 N/mm^2. The result further showed that the split tensile results varied between 18% - 19% of its compressive strength result. This indicates that the materials and the mix ratios are suitable and building blocks for residential low rising buildings and as partition slaps for high rising buildings. The lightweight property makes it highly suitable for large scale application in high rising structures as internal partition slaps only.

Table 3. Residuals for tensile strength (Scheffe's pseudo component model)

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	1.244912	2.469955	1.747486	.2656287	20
Residual	-.1901037	.4992848	.0062411	.1389019	20
Std. Predicted Value	-1.892	2.720	.000	1.000	20
Std. Residual	-.992	2.605	.033	.725	20

Table 4. Mix ratios, laboratory and model split tensile strength for pseudo components model

S/N	X1	X2	X3	X4	X1 * X2	X1 * X3	X1 * X4	X2 * X3	X2 * X4	X3 * X4	Experiment strength	Model strength
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.47	2.47
2	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.99	1.98
3	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.61	1.61
4	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	1.36	1.32
5	0.500	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000	1.98	2.06
6	0.500	0.000	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.000	1.72	2.25
7	0.500	0.000	0.000	0.500	0.000	0.000	0.250	0.000	0.000	0.000	1.81	1.80
8	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.250	0.000	0.000	1.50	1.90
9	0.000	0.500	0.000	0.500	0.000	0.000	0.000	0.000	0.250	0.000	1.71	1.75
10	0.000	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.000	0.250	1.42	1.54
11	0.500	0.250	0.250	0.000	0.125	0.125	0.000	0.063	0.000	0.000	2.04	2.18
12	0.250	0.250	0.250	0.250	0.063	0.063	0.063	0.063	0.063	0.063	1.78	1.90
13	0.000	0.250	0.000	0.750	0.000	0.000	0.000	0.000	0.188	0.000	1.49	1.56
14	0.500	0.000	0.250	0.250	0.000	0.125	0.125	0.000	0.000	0.063	1.83	2.04
15	0.500	0.250	0.000	0.250	0.125	0.000	0.125	0.000	0.063	0.000	2.00	1.95
16	0.000	0.250	0.750	0.000	0.000	0.000	0.000	0.188	0.000	0.000	1.64	1.78
17	0.000	0.250	0.250	0.250	0.000	0.000	0.000	0.063	0.063	0.063	1.74	1.30
18	0.250	0.125	0.500	0.125	0.031	0.125	0.031	0.063	0.016	0.063	1.80	1.96
19	0.250	0.250	0.000	0.500	0.063	0.000	0.125	0.000	0.125	0.000	1.65	1.73
20	0.125	0.125	0.250	0.500	0.016	0.031	0.063	0.031	0.063	0.125	1.53	1.68

Source: Author's computation, 2020

Table 5. Optimization result for split tensile strength using the Scheffe's Pseudo component model

SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm^2	SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm^2
1	0.482	1	1.82	3.28	5.01	70	0.458	1	1.88	3.16	5.08
2	0.48	1	1.83	3.28	5.01	71	0.46	1	1.85	3.1	5.09
3	0.478	1	1.84	3.28	5.01	72	0.46	1	1.84	3.08	5.09
4	0.479	1	1.81	3.22	5.02	73	0.46	1	1.83	3.06	5.08
5	0.475	1	1.85	3.28	5.01	74	0.462	1	1.81	3.02	5.09
6	0.477	1	1.82	3.22	5.02	75	0.462	1	1.8	3	5.09
7	0.473	1	1.86	3.28	5.01	76	0.452	1	1.97	3.32	5.06
8	0.474	1	1.85	3.26	5.02	77	0.453	1	1.96	3.3	5.07
9	0.475	1	1.83	3.22	5.03	78	0.453	1	1.95	3.28	5.07
10	0.475	1	1.82	3.2	5.03	79	0.453	1	1.94	3.26	5.08
11	0.476	1	1.8	3.16	5.04	80	0.454	1	1.93	3.24	5.08
12	0.471	1	1.87	3.28	5.02	81	0.455	1	1.91	3.2	5.09
13	0.471	1	1.86	3.26	5.02	82	0.455	1	1.9	3.18	5.09
14	0.473	1	1.84	3.22	5.03	83	0.456	1	1.89	3.16	5.09
15	0.473	1	1.83	3.2	5.03	84	0.456	1	1.88	3.14	5.09
16	0.474	1	1.81	3.16	5.04	85	0.457	1	1.87	3.12	5.09
17	0.475	1	1.8	3.14	5.04	86	0.457	1	1.86	3.1	5.08
18	0.469	1	1.88	3.28	5.02	87	0.458	1	1.84	3.06	5.09
19	0.469	1	1.87	3.26	5.03	88	0.459	1	1.83	3.04	5.09
20	0.471	1	1.84	3.2	5.04	89	0.459	1	1.82	3.02	5.08
21	0.472	1	1.81	3.14	5.05	90	0.46	1	1.8	2.98	5.09
22	0.467	1	1.89	3.28	5.03	91	0.45	1	1.98	3.32	5.08
23	0.467	1	1.88	3.26	5.03	92	0.45	1	1.97	3.3	5.08
24	0.469	1	1.85	3.2	5.04	93	0.451	1	1.96	3.28	5.08
25	0.469	1	1.84	3.18	5.05	94	0.451	1	1.95	3.26	5.09
26	0.47	1	1.82	3.14	5.06	95	0.452	1	1.94	3.24	5.09
27	0.471	1	1.81	3.12	5.06	96	0.452	1	1.93	3.22	5.09
28	0.472	1	1.79	3.08	5.06	97	0.452	1	1.92	3.2	5.09
29	0.464	1	1.9	3.28	5.03	98	0.453	1	1.91	3.18	5.09
30	0.465	1	1.89	3.26	5.04	99	0.453	1	1.9	3.16	5.09
31	0.465	1	1.88	3.24	5.04	100	0.454	1	1.88	3.12	5.1
32	0.466	1	1.86	3.2	5.05	101	0.455	1	1.87	3.1	5.09

SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm^2	SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm^2
33	0.467	1	1.85	3.18	5.05	102	0.455	1	1.86	3.08	5.09
34	0.467	1	1.84	3.16	5.06	103	0.456	1	1.85	3.06	5.09
35	0.468	1	1.82	3.12	5.06	104	0.456	1	1.84	3.04	5.08
36	0.469	1	1.81	3.1	5.06	105	0.457	1	1.83	3.02	5.09
37	0.47	1	1.79	3.06	5.07	106	0.457	1	1.82	3	5.09
38	0.462	1	1.91	3.28	5.04	107	0.458	1	1.81	2.98	5.08
39	0.463	1	1.9	3.26	5.04	108	0.448	1	1.99	3.32	5.09
40	0.463	1	1.89	3.24	5.05	109	0.448	1	1.98	3.3	5.09
41	0.465	1	1.86	3.18	5.06	110	0.449	1	1.97	3.28	5.1
42	0.465	1	1.85	3.16	5.06	111	0.449	1	1.96	3.26	5.1
43	0.467	1	1.82	3.1	5.07	112	0.449	1	1.95	3.24	5.1
44	0.467	1	1.81	3.08	5.07	113	0.45	1	1.94	3.22	5.1
45	0.46	1	1.92	3.28	5.05	114	0.45	1	1.93	3.2	5.1
46	0.46	1	1.91	3.26	5.05	115	0.451	1	1.92	3.18	5.1
47	0.461	1	1.9	3.24	5.06	116	0.451	1	1.91	3.16	5.1
48	0.461	1	1.89	3.22	5.06	117	0.451	1	1.9	3.14	5.1
49	0.463	1	1.86	3.16	5.07	118	0.452	1	1.89	3.12	5.09
50	0.463	1	1.85	3.14	5.07	119	0.452	1	1.88	3.1	5.09
51	0.464	1	1.84	3.12	5.07	120	0.453	1	1.87	3.08	5.08
52	0.465	1	1.82	3.08	5.08	121	0.453	1	1.86	3.06	5.07
53	0.465	1	1.81	3.06	5.08	122	0.454	1	1.85	3.04	5.09
54	0.466	1	1.79	3.02	5.08	123	0.454	1	1.84	3.02	5.08
55	0.458	1	1.92	3.26	5.06	124	0.455	1	1.83	3	5.07
56	0.459	1	1.91	3.24	5.07	125	0.455	1	1.82	2.98	5.07
57	0.459	1	1.9	3.22	5.07						
58	0.459	1	1.89	3.2	5.07						
59	0.46	1	1.88	3.18	5.07						
60	0.461	1	1.86	3.14	5.08						
61	0.461	1	1.85	3.12	5.08						
62	0.462	1	1.84	3.1	5.08						
63	0.463	1	1.82	3.06	5.09						
64	0.463	1	1.81	3.04	5.08						
65	0.454	1	1.96	3.32	5.05						
66	0.456	1	1.92	3.24	5.07						

SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm^2	SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm^2
67	0.457	1	1.91	3.22	5.08						
68	0.457	1	1.9	3.2	5.08						
69	0.458	1	1.89	3.18	5.08						

Source: Author's research, 2020

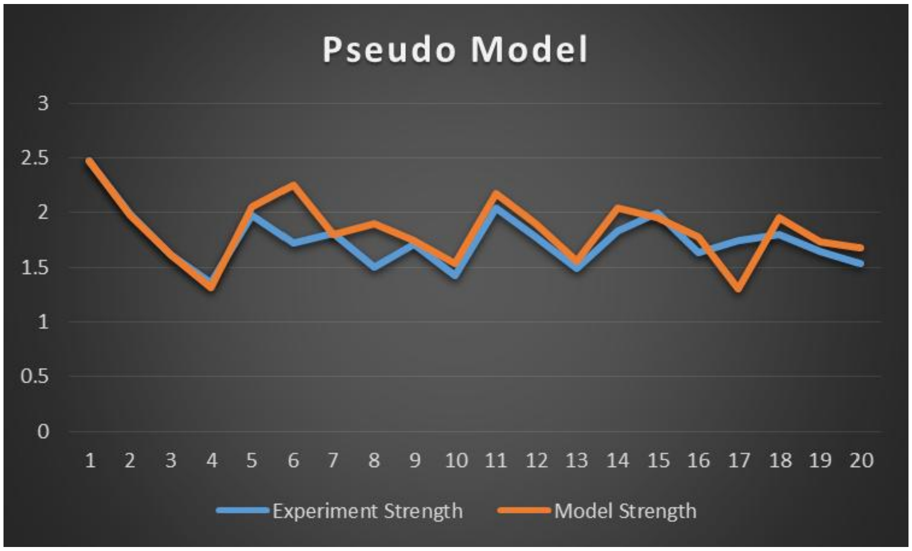

Fig. 1. Graph showing the experiment result against the model for the Scheffe Pseudo Component Model

4.2 Scheffe's Component Proportion Model

The estimated regression coefficients for the components proportion model are given in Table 6 while the Anova table is presented in Table 7.

4.2.1 Model equation

From Table 6 the estimated coefficients for the Scheffe's second degree polynomial are given as:

$$\beta_1 = 0, \quad \beta_2 = 0, \quad \beta_3 = 0, \quad \beta_4 = 0, \quad \beta_{12} = 1.069, \quad \beta_{13} = 71.589, \quad \beta_{14} = 13.307, \quad \beta_{23} = -5.313, \quad \beta_{24} = 0, \quad \beta_{34} = 2.187$$

If we let the components' proportions of water, cement, sand and quarry dust be represented respectively by Z_1, Z_2, Z_3 and Z_4, then the model equation in terms of components' proportions is:

$$Y = 1.069Z_1Z_2 + 71.589Z_1Z_3 + 13.307Z_1Z_4 - 5.313Z_2Z_3 + 2.187Z_3Z_4 \qquad \text{(xi)}$$

This model suggests that components $Z_1, Z_3, \; and \; Z_4$ themselves contribute nothing to the response of the mixture. Similarly, components $Z_2Z_3, \; and \; Z_2Z_4$ do not also contribute to the response.

4.2.2 Test for lack-of-fit

Table 8 shows that there is insignificant lack-of-fit, the *p-value* for lack-of-fit being 0.00 which is less than 0.05. The conclusion, therefore, is that Equation (*xi*) is adequate for predicting the 28th day split tensile strength of expanded polystyrene concrete. The other statistics in Table 7, lend credence to the adequacy of the model.

Table 6. Estimated regression coefficients for tensile strength

Model		Unstandardized coefficients		Standardized coefficients	t	Sig.
		B	Std. Error	Beta		
1	Z1 * Z2	1.069	55.292	.005	.019	.985
	Z1 * Z3	71.589	71.692	.691	.999	.334
	Z1 * Z4	13.307	16.581	.234	.803	.435
	Z2 * Z3	-5.313	26.178	-.111	-.203	.842
	Z3 * Z4	2.187	2.529	.186	.865	.401

(Scheffe's Component proportion model)

Table 7. Analysis of Variance for Tensile strength (Scheffe's component proportion model)

Model		Sum of Squares	df	Mean square	F	Sig.
1	Regression	62.624	5	12.525	1185.356	.000[c]
	Residual	.158	15	.011		
	Total	62.782[d]	20			

Table 8. Residuals for tensile strength (Scheffe's component proportion model)

	Minimum	Maximum	Mean	Std. deviation	N
Predicted Value	1.394858	2.458852	1.753767	.2416657	20
Residual	-.2293126	.1037125	-.0000401	.0913332	20
Std. Predicted Value	-1.485	2.918	.000	1.000	20
Std. Residual	-2.231	1.009	.000	.889	20

4.2.3 Experiment versus model tensile strength (component proportion model)

Results in Table 9 shows the mixes, laboratory as well as the model values for the split tensile strength of polystyrene lightweight concrete using the Scheffe's component proportion model. The modelled result shows that the model was able to significantly predict the tensile strength.

4.2.4 Optimization result for component proportion model

Data in Table 10 shows the optimized split tensile strength result for the component proportion model. The results we generated by the optimizer program using the simplex lattice design response data from Appendix1a & b, and the real ratios as contained in Appendix2. Just like the pseudo component model, the results from this model also yielded an average optimized split tensile strength of about 5.1 N/mm^2. The result further showed that the split tensile results varied between 18% - 19% of its compressive strength result. This also indicates that these materials and the mix ratios are suitable for making building blocks for residential low rising buildings and as partition slaps for high rising buildings. The lightweight property makes it highly suitable for large scale application in high rising structures as internal partition slaps only. Simply put, this model has proven effective in optimizing the split tensile strength property of polystyrene lightweight concrete.

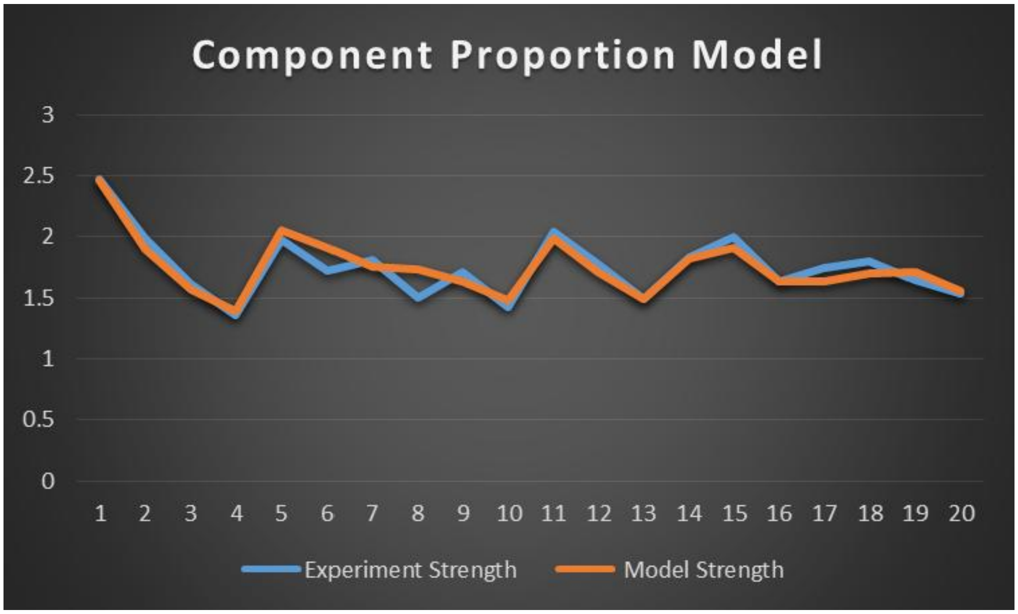

Fig. 2. Graph showing the experiment result against the model for the Scheffe's component proportion model

Table 9. Mix ratios, laboratory and model split tensile strength for component proportion model

S/N	Z_1	Z_2	Z_3	Z_4	$Z_1 * Z_2$	$Z_1 * Z_3$	$Z_1 * Z_4$	$Z_2 * Z_3$	$Z_2 * Z_4$	$Z_3 * Z_4$	Experiment strength	Model strength
1	0.091	0.202	0.303	0.404	0.018	0.028	0.037	0.061	0.082	0.122	2.47	2.46
2	0.067	0.133	0.267	0.533	0.009	0.018	0.036	0.036	0.071	0.142	1.99	1.90
3	0.051	0.112	0.279	0.558	0.006	0.014	0.029	0.031	0.062	0.156	1.61	1.57
4	0.042	0.096	0.287	0.575	0.004	0.012	0.024	0.028	0.055	0.165	1.36	1.39
5	0.076	0.161	0.281	0.482	0.012	0.021	0.037	0.045	0.077	0.135	1.98	2.06
6	0.065	0.144	0.288	0.503	0.009	0.019	0.033	0.041	0.072	0.145	1.72	1.91
7	0.058	0.13	0.292	0.52	0.008	0.017	0.03	0.038	0.068	0.152	1.81	1.76
8	0.058	0.122	0.273	0.547	0.007	0.016	0.032	0.033	0.066	0.149	1.5	1.73
9	0.052	0.111	0.279	0.557	0.006	0.015	0.029	0.031	0.062	0.155	1.71	1.64
10	0.046	0.103	0.284	0.567	0.005	0.013	0.026	0.029	0.058	0.161	1.42	1.48
11	0.071	0.152	0.285	0.493	0.011	0.02	0.035	0.043	0.075	0.14	2.04	1.99
12	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.035	0.067	0.151	1.78	1.71
13	0.047	0.103	0.283	0.567	0.005	0.013	0.027	0.029	0.058	0.161	1.49	1.49
14	0.061	0.137	0.29	0.512	0.008	0.018	0.031	0.04	0.07	0.149	1.83	1.82
15	0.066	0.144	0.287	0.503	0.009	0.019	0.033	0.041	0.072	0.145	2	1.91
16	0.055	0.116	0.276	0.553	0.006	0.015	0.03	0.032	0.064	0.153	1.64	1.64
17	0.052	0.112	0.279	0.558	0.006	0.015	0.029	0.031	0.062	0.155	1.74	1.64
18	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.036	0.067	0.151	1.8	1.70
19	0.055	0.12	0.285	0.54	0.007	0.016	0.03	0.034	0.065	0.154	1.65	1.71
20	0.05	0.111	0.284	0.555	0.006	0.014	0.028	0.032	0.062	0.158	1.53	1.56

Source: Author’s research, 2020

Table 10. Optimization result for split tensile strength using the Scheffe's component proportion model

SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm2	SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm2
1	0.482	1	1.85	3.360	5.12	74	0.46	1	1.86	3.140	5.11
2	0.48	1	1.85	3.340	5.11	75	0.46	1	1.87	3.160	5.10
3	0.48	1	1.85	3.340	5.12	76	0.46	1	1.87	3.160	5.10
4	0.48	1	1.84	3.320	5.13	77	0.461	1	1.87	3.160	5.10
5	0.478	1	1.85	3.320	5.12	78	0.461	1	1.88	3.180	5.10
6	0.479	1	1.85	3.320	5.12	79	0.461	1	1.87	3.160	5.11
7	0.476	1	1.85	3.300	5.11	80	0.461	1	1.87	3.160	5.11
8	0.476	1	1.85	3.300	5.12	81	0.457	1	1.86	3.120	5.10
9	0.477	1	1.85	3.300	5.12	82	0.457	1	1.87	3.140	5.09
10	0.474	1	1.85	3.280	5.11	83	0.458	1	1.86	3.120	5.10
11	0.475	1	1.85	3.280	5.12	84	0.458	1	1.87	3.140	5.09
12	0.475	1	1.86	3.300	5.11	85	0.458	1	1.86	3.120	5.10
13	0.475	1	1.85	3.280	5.12	86	0.458	1	1.87	3.140	5.10
14	0.475	1	1.85	3.280	5.12	87	0.458	1	1.87	3.140	5.10
15	0.472	1	1.85	3.260	5.11	88	0.458	1	1.88	3.160	5.09
16	0.472	1	1.86	3.280	5.11	89	0.459	1	1.87	3.140	5.10
17	0.473	1	1.85	3.260	5.12	90	0.459	1	1.88	3.160	5.10
18	0.473	1	1.86	3.280	5.11	91	0.459	1	1.87	3.140	5.11
19	0.473	1	1.85	3.260	5.12	92	0.459	1	1.88	3.160	5.10
20	0.473	1	1.86	3.280	5.11	93	0.46	1	1.87	3.140	5.11
21	0.47	1	1.86	3.260	5.10	94	0.455	1	1.87	3.120	5.09
22	0.471	1	1.85	3.240	5.11	95	0.455	1	1.86	3.100	5.10
23	0.471	1	1.86	3.260	5.11	96	0.455	1	1.87	3.120	5.09
24	0.471	1	1.85	3.240	5.12	97	0.456	1	1.86	3.100	5.10
25	0.471	1	1.86	3.260	5.11	98	0.456	1	1.87	3.120	5.09
26	0.471	1	1.86	3.260	5.11	99	0.456	1	1.87	3.120	5.10
27	0.468	1	1.86	3.240	5.10	100	0.457	1	1.87	3.120	5.10
28	0.468	1	1.86	3.240	5.10	101	0.457	1	1.88	3.140	5.09
29	0.469	1	1.85	3.220	5.11	102	0.457	1	1.87	3.120	5.10
30	0.469	1	1.86	3.240	5.11	103	0.457	1	1.88	3.140	5.10
31	0.469	1	1.85	3.220	5.12	104	0.457	1	1.87	3.120	5.11
32	0.469	1	1.86	3.240	5.11	105	0.457	1	1.88	3.140	5.10

SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm2	SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm2
33	0.47	1	1.86	3.240	5.11	106	0.453	1	1.87	3.100	5.09
34	0.47	1	1.86	3.240	5.11	107	0.454	1	1.86	3.080	5.10
35	0.466	1	1.86	3.220	5.10	108	0.454	1	1.87	3.100	5.09
36	0.467	1	1.86	3.220	5.10	109	0.454	1	1.87	3.100	5.09
37	0.467	1	1.85	3.200	5.11	110	0.454	1	1.87	3.100	5.10
38	0.467	1	1.86	3.220	5.11	111	0.454	1	1.88	3.120	5.09
39	0.467	1	1.86	3.220	5.11	112	0.455	1	1.87	3.100	5.10
40	0.467	1	1.87	3.240	5.10	113	0.455	1	1.88	3.120	5.09
41	0.468	1	1.86	3.220	5.11	114	0.455	1	1.87	3.100	5.10
42	0.468	1	1.87	3.240	5.10	115	0.455	1	1.88	3.120	5.09
43	0.468	1	1.86	3.220	5.11	116	0.456	1	1.88	3.120	5.10
44	0.464	1	1.86	3.200	5.10	117	0.456	1	1.88	3.120	5.10
45	0.465	1	1.86	3.200	5.10	118	0.452	1	1.87	3.080	5.09
46	0.465	1	1.86	3.200	5.11	119	0.452	1	1.88	3.100	5.09
47	0.466	1	1.86	3.200	5.11	120	0.453	1	1.87	3.080	5.10
48	0.466	1	1.87	3.220	5.10	121	0.453	1	1.88	3.100	5.09
49	0.466	1	1.86	3.200	5.11	122	0.453	1	1.87	3.080	5.10
50	0.466	1	1.87	3.220	5.10	123	0.453	1	1.88	3.100	5.09
51	0.466	1	1.86	3.200	5.11	124	0.453	1	1.88	3.100	5.09
52	0.463	1	1.86	3.180	5.10	125	0.454	1	1.88	3.100	5.10
53	0.463	1	1.86	3.180	5.10	126	0.454	1	1.89	3.120	5.09
54	0.463	1	1.86	3.180	5.11	127	0.454	1	1.88	3.100	5.10
55	0.463	1	1.87	3.200	5.10	128	0.451	1	1.88	3.080	5.09
56	0.464	1	1.86	3.180	5.11	129	0.451	1	1.87	3.060	5.10
57	0.464	1	1.87	3.200	5.10	130	0.451	1	1.88	3.080	5.09
58	0.464	1	1.86	3.180	5.11	131	0.452	1	1.88	3.080	5.09
59	0.464	1	1.87	3.200	5.10	132	0.452	1	1.88	3.080	5.10
60	0.465	1	1.87	3.200	5.11	133	0.452	1	1.89	3.100	5.09
61	0.461	1	1.86	3.160	5.10	134	0.452	1	1.88	3.080	5.10
62	0.461	1	1.86	3.160	5.10	135	0.453	1	1.88	3.080	5.10
63	0.461	1	1.87	3.180	5.10	136	0.45	1	1.88	3.060	5.09
64	0.462	1	1.86	3.160	5.11	137	0.45	1	1.89	3.080	5.09
65	0.462	1	1.87	3.180	5.10	138	0.45	1	1.88	3.060	5.10
66	0.462	1	1.86	3.160	5.11	139	0.45	1	1.89	3.080	5.09

SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm2	SN	Water %	Cement %	Sand %	Quarry %	S T S N/mm2
67	0.462	1	1.87	3.180	5.10	140	0.451	1	1.88	3.060	5.10
68	0.462	1	1.87	3.180	5.10	141	0.451	1	1.89	3.080	5.09
69	0.463	1	1.87	3.180	5.11	142	0.448	1	1.89	3.060	5.09
70	0.459	1	1.86	3.140	5.10	143	0.449	1	1.89	3.060	5.09
71	0.459	1	1.87	3.160	5.09	144	0.447	1	1.89	3.040	5.09
72	0.459	1	1.86	3.140	5.10						
73	0.459	1	1.87	3.160	5.09						

Source: Author's research, 2020

5. CONCLUSION

This study have shown that mathematical models can accurately predict the tensile strength of polystyrene lightweight concrete vis-à-vis strength properties of polystyrene lightweight concrete. Results obtained are adequate for residential applications with near commercial applications, hence the need for further studies. This therefore implies the need for changes in policy regarding the use of polystyrene as a concrete aggregate as a means to managing the polystyrene waste generated. It will also contribute significantly to realizing the SDG 11 which aims at developing sustainable communities

6. RECOMMENDATION

It is recommended that government should begin to develop low cost housing estates using this method to serve as models for sustainable housing development. To achieve this, modalities should be put in place to for efficient disposal of polystyrene waste. Lastly, although, this study has corroborated existing claims that mathematical models are accurate and cost effective, it is however recommended that further studies should be conducted to improve on the findings of this study.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Chen B, Liu J. Mechanical properties of polymer-modified concretes containing expanded polystyrene beads. Construction and Building Materials. 2007;21(1):7-11.
2. Kumar A, Kumari Y, Kumar LJ, Kumar V. Structural and morphological study of poly (methyl methacrylate)/polyaniline composite membranes, Materials Today Proceeding. 2020;31:674-678.
3. Chen B, Liu N. A novel lightweight concrete-fabrication and its thermal and mechanical properties. Construction and building materials. 2013;44:691-698.
4. Kan A, Demirboğa R. A novel material for lightweight concrete production. Cement and Concrete Composites, 2009;31(7):489-495.
5. Gawale R, Mishra S, Sambare H, Kothari J, Patil M. Light Weight Concrete by Using EPS Beads. International Journal of Innovative Research in Science, Engineering and Technology. 2016; 2(03):470-476.
6. Tamut T, Prabhu R, Venkataramana K, Yaragal SC. Partial replacement of coarse aggregates by expanded polystyrene beads in concrete. International Journal of Research in Engineering and Technology. 2014;3(2):238-241.
7. Kumari Y, Kumar L, Kumar A, Kumar M, Swami KC, Awasthi K. Structural and Morphological Study of PS-TiO2 Nanocomposite Membrane, Macromolecular Symposia. 2015;357:200–205
8. Kumar A, Jangir LK, Kumari Y, Kumar M, Kumar V, Awasthi K. Optical and Structural Study of Polyaniline/Polystyrene Composite Films, Macromolecular Symposia. 2015;357:229–234.
9. Ubi SE, Okafor FO, Mama BO. Optimisation of compressive strength of polystyrene Lightweight Concrete Using Scheffe" s Pseudo and Component Proportion Models. SSRG International Journal of Civil Engineering (SSRG-IJCE). 2020a;7(6):21-35.
10. Kuhail Z, Shihada S. Mechanical properties of polystyrene-lightweight concrete. IUG Journal of Natural Studies. 2015;11(2).
11. UN Habitat (2020). Global housing strategy making housing affordable for all. Available:https://unhabitat.org/programme/global-housing-strategy-making-housing-affordable-for-all. Accessed online (15/11/2020).
12. Kumar LJ, Kumari Y, Kumar A, Kumar M, Awasthi K. Structural and morphological study of PS-ZnO Nanocomposite Membrane, Macromolecular Symposia. 2015;357:218–222
13. Kumar A, Kumar V, Awasthi K. Polyaniline-Carbon Nanotube Composites: Preparation Methods, Properties and Applications, Polymer-Plastics Technology and Engineering. 2017;57(2): 70-97.
14. Cornell J. Experiments with Mixtures: Designs, Models and the Analysis of Mixture Data. 3ed. John Wiley and Sons Inc. New York; 2002.
15. Piepel GF, Redgate T. A mixture experiment analysis of the hald cement data. The American Statistician. 1998;52:23–30.

16. Goelz JCG. Systematic experimental designs for mixed-species plantings. Native plant Journal, 2001;2(1).
17. Ubi SE, Okafor FO, Mama BO, Agbor RB. Prediction of Flexural Strength of Expanded Polystrene Lightweight Concrete using Scheffe's Model International Journal of Advanced Research in Engineering and Technology; 2020b.
18. Ubi, SE, Nkra PO, Agbor RB, Okafor FO. Mathematical Model For Optimization of Modulus of Elasticity of Polystyrene Lightweight Concrete Using Scheffe's Model, SSRG International Journal of Civil Engineering. 2020c;7(10):16-30.
19. Scheffe, H. Experiment with mixtures. Journal of Royal Statistics Society. Series B. 1958;20(2):344–360.
20. EN, B. 206: 2013. Concrete. Specification, performance, production and conformity; 2013.
21. Nigerian Industrial Standard. (NIS 87:2004). Standard for sandcrete blocks, standards organisation of Nigeria, Lagos, Nigeria; 2004.
22. BS 1881: Part 117: 1983 British Standard Testing Concrete Part 117. Method for determination of tensile splitting strength; 1983.
23. IS: 5816-1970 ‘Splitting Tensile Strength of Concrete method of Test’. Indian Standard.
24. Astm C. Standard test method for splitting tensile strength of cylindrical concrete specimens. C496/C496M-11; 2011.

APPENDIX

Appendix 1a. Actual (Zi) and pseudo (xi) components for Scheffe's (4, 2) simplex lattice

S/N	X_1	X_2	X_3	X_4	Response	R_1	R_2	R_3	R_4
1.	1	0	0	0	Y_1	0.45	0.50	0.46	0.44
2.	0	1	0	0	Y_2	1	1	1	1
3.	0	0	1	0	Y_3	1.5	2.0	2.5	3.0
4.	0	0	0	1	Y_4	3	4.0	5.0	6.0
5.	½	½	0	0	Y_{12}	0.475	1	2.75	3.5
6.	½	0	½	0	Y_{13}	0.455	1	2.0	5.0
7.	½	0	0	½	Y_{14}	0.445	1	2.25	4.5
8.	0	½	½	0	Y_{23}	0.48	1	2.25	4.5
9.	0	½	0	½	Y_{24}	0.47	1	2.5	4.5
10.	0	0	½	½	Y_{34}	0.45	1	2.75	5.5

Appendix 1b. Control points actual (xi) and pseudo (zi) componentsfor Scheffe's (4, 2) simplex lattice

S/N	X_1	X_2	X_3	X_4	Response	R_1	R_2	R_3	R_4
11.	½	$^1/_4$	¼	0	C_1	0.465	1	1.88	3.75
12.	$^1/_4$	$^1/_4$	¼	$^1/_4$	C_2	0.463	1	2.25	4.5
13.	0	$^1/_4$	0	$^3/_4$	C_3	0.46	1	2.63	5.5
14.	½	0	¼	$^1/_4$	C_4	0.48	1	2.13	4.25
15.	½	$^1/_4$	0	$^1/_4$	C_5	0.46	1	2.0	4.0
16.	0	$^1/_4$	¾	0	C_6	0.47	1	2.38	4.75
17.	0	½	¼	$^1/_4$	C_7	0.475	1	2.13	4.75
18.	$^1/_4$	$^1/_8$	½	$^1/_8$	C_8	0.46	1	2.25	4.50
19.	$^1/_4$	$^1/_4$	0	½	C_9	0.458	1	2.38	4.75
20.	$^1/_8$	$^1/_8$	¼	½	C_{10}	0.454	1	2.56	5.13

Appendix 2. Scheffe's {4,2} lattice simplex ratios Vs Eperimential, Model-1 and Model-2 results

S/N	X_1	X_2	X_3	X_4	X_1 * X_2	X_1 * X_3	X_1 * X_4	X_2 * X_3	X_2 * X_4	X_3 * X_4	R_1	R_2	R_3	R_4	Z_1	Z_2	Z_3	Z_4	Z_1 * Z_2	Z_1 * Z_3	Z_1 * Z_4	Z_2 * Z_3	Z_2 * Z_4	Z_3 * Z_4	STS experiment	STS Model1	STS Model2
1	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.45	1.00	1.50	2.00	0.091	0.202	0.303	0.404	0.018	0.028	0.037	0.061	0.082	0.122	2.47	2.47	2.42
2	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.50	1.00	2.00	4.00	0.067	0.133	0.267	0.533	0.009	0.018	0.036	0.036	0.071	0.142	1.99	1.98	1.88
3	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.46	1.00	2.50	5.00	0.051	0.112	0.279	0.558	0.006	0.014	0.029	0.031	0.062	0.156	1.61	1.61	1.59
4	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.44	1.00	3.00	6.00	0.042	0.096	0.287	0.575	0.004	0.012	0.024	0.028	0.055	0.165	1.36	1.32	1.41
5	0.500	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.000	0.48	1.00	1.75	3.00	0.076	0.161	0.281	0.482	0.012	0.021	0.037	0.045	0.077	0.135	1.98	2.06	2.09
6	0.500	0.000	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.000	0.46	1.00	2.00	3.50	0.065	0.144	0.288	0.503	0.009	0.019	0.033	0.041	0.072	0.145	1.72	1.83	1.89
7	0.500	0.000	0.000	0.500	0.000	0.000	0.250	0.000	0.000	0.000	0.45	1.00	2.25	4.00	0.058	0.130	0.292	0.520	0.008	0.017	0.030	0.038	0.068	0.152	1.81	1.80	1.75
8	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.250	0.000	0.000	0.48	1.00	2.25	4.50	0.058	0.122	0.273	0.547	0.007	0.016	0.032	0.033	0.066	0.149	1.5	1.69	1.72
9	0.000	0.500	0.000	0.500	0.000	0.000	0.000	0.000	0.250	0.000	0.47	1.00	2.50	5.00	0.052	0.111	0.279	0.557	0.006	0.015	0.029	0.031	0.062	0.155	1.71	1.75	1.62
10	0.000	0.000	0.500	0.500	0.000	0.000	0.000	0.000	0.000	0.250	0.45	1.00	2.75	5.50	0.046	0.103	0.284	0.567	0.005	0.013	0.026	0.029	0.058	0.161	1.42	1.54	1.49
11	0.500	0.250	0.250	0.000	0.125	0.125	0.000	0.063	0.000	0.000	0.47	1.00	1.88	3.25	0.071	0.152	0.285	0.493	0.011	0.020	0.035	0.043	0.075	0.140	2.04	1.92	1.99
12	0.250	0.250	0.250	0.250	0.063	0.063	0.063	0.063	0.063	0.063	0.46	1.00	2.25	4.25	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.035	0.067	0.151	1.78	1.74	1.74
13	0.000	0.250	0.000	0.750	0.000	0.000	0.000	0.000	0.188	0.000	0.46	1.00	2.75	5.50	0.047	0.103	0.283	0.567	0.005	0.013	0.027	0.029	0.058	0.161	1.49	1.56	1.51
14	0.500	0.000	0.250	0.250	0.000	0.125	0.125	0.000	0.000	0.063	0.45	1.00	2.13	3.75	0.061	0.137	0.290	0.512	0.008	0.018	0.031	0.040	0.070	0.149	1.83	1.83	1.82
15	0.500	0.250	0.000	0.250	0.125	0.000	0.125	0.000	0.063	0.000	0.46	1.00	2.00	3.50	0.066	0.144	0.287	0.503	0.009	0.019	0.033	0.041	0.072	0.145	2	1.95	1.91
16	0.000	0.250	0.750	0.000	0.000	0.000	0.000	0.188	0.000	0.000	0.47	1.00	2.38	4.75	0.055	0.116	0.276	0.553	0.006	0.015	0.030	0.032	0.064	0.153	1.64	1.62	1.65
17	0.000	0.250	0.250	0.250	0.000	0.000	0.000	0.063	0.063	0.063	0.35	0.75	1.88	3.75	0.052	0.112	0.279	0.558	0.006	0.015	0.029	0.031	0.062	0.155	1.74	1.25	1.61
18	0.250	0.125	0.500	0.125	0.031	0.125	0.031	0.063	0.016	0.063	0.46	1.00	2.25	4.25	0.058	0.126	0.283	0.534	0.007	0.016	0.031	0.036	0.067	0.151	1.8	1.69	1.73
19	0.250	0.250	0.000	0.500	0.063	0.000	0.125	0.000	0.125	0.000	0.46	1.00	2.38	4.50	0.055	0.120	0.285	0.540	0.007	0.016	0.030	0.034	0.065	0.154	1.65	1.73	1.68
20	0.125	0.125	0.250	0.500	0.016	0.031	0.063	0.031	0.063	0.125	0.45	1.00	2.56	5.00	0.050	0.111	0.284	0.555	0.006	0.014	0.028	0.032	0.062	0.158	1.53	1.61	1.58

Source: Author's research work

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle4.com/review-history/64463