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A Mathematical Model for Plato's Theory of Forms

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Abstract

Aims/ objectives: In this article we construct a mathematical/topological framework for comprehending fundamental concepts in Plato's theory of Forms; specifically the dual processes of:
1. The participation/partaking-methexis of the many particulars predicated as F to the Form-essence F, according to their degree of participation to it.
2. The presence-parousia of the Form-essence F to the particulars predicated as F, in analogy to their degree of participation to F as in 1.

The theoretical foundation of our model is primarily based on a combination of both the Approximationist and Predicationalist approaches for Plato's theory of Forms, taking into account the degree of participation of the particulars to the Form, that are predicated to. In constructing our model we assume that there exists exactly one Form corresponding to every predicate that has a Form (Plato's 'uniqueness thesis'), and to support our main theses we analyze textual evidence from various Platonic works. The mathematical model is founded on the dual notions of projective and inductive topologies, and their projective and inductive limits respectively.

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1 Introduction

In this section we introduce the problem, and in the second section we present in brief Plato's theory of Forms, as well as the foundational background of our main arguments. In section three we present our central theses and the arguments that culminate to the Main Claim. In section four we defend our central thesis and we provide a 'proof' of the Main Claim. Section five analyzes the mathematical framework and the specific topologies within which our arguments and the main thesis are founded on and conceptualized. In the last section, summarizing the work done in the previous sections, we briefly present the main conclusions of this work.

The main goal of this article is to provide a mathematical/topological model that serves as a solid framework for comprehending the dual processes of:
1 The participation/partaking-methexis of the many particulars predicated as F to the Form F ('identified' with its essence), according to their degree of participation to it.
2 The presence-parousia of the Form-essence F to the many particulars predicated as F, in analogy to their degree of participation to F as in 1.

Plato's Theory of Forms should be also viewed as a theory of judgment and predication. Judging involves consulting Forms: To judge that a sensible particular x is F, or is predicated as F, is to consult the Form of F-ness and to perceive x as being sufficiently like F-ness to qualify for the predicate F. On this, Plato in *Phaedo* $102b1-2$ states that 'each of the Forms εἴδη (abstract qualities) exists and that other things which participate in these get their names from them'.

Throughout this article we assume the uniquness thesis. That is:

'There exists exactly one Form corresponding to every predicate that has a Form'.

As we shall see later this F-Form is not only unique, but also is identified to its essence F-ness. (For a different version of the uniqueness thesis and an extensive discussion the reader should consult G. Fine [1] pp. $117, 189-190, 205, 239, 304$.)

Plato's intention in defending the uniqueness of a Form per predicate was clearly introduced in the 'Third Bed Argument', TBA, *Republic* $597c-d$. (A study and an analysis of this is presented in G. Fine [1] pp. $231-238$)[1].

In developing and defending our arguments, we shall be consistent with the interpretation of the presence of a property in a thing/particular, as well as the recurrence of a single property in different things/particulars. According to Scaltsas [2], the things are F by participating in a Form F-ness is the answer to two different questions that Plato implies in *Phaedo* $100c9-d8$, *Parm.* $128e6-129a4$, $130e5-131a2$, as well as earlier in *Meno* $72c$. The first is 'Why is a thing F?' Thus, the first question concerns the predication of F-ness. In *Phaedo* $100c9-d8$ the Forms are introduced as the causes of things being F[2]. The second question is 'Why are different things similar?' This question that appears clearly in *Parm.* $128e6-129a4, 130e5-131a2$, concerns recurrence of a single property in different things and considers the quality identity with respect to F-ness.

In the construction of our mathematical model we consider, according to the 'Approximationist' approach to the Forms, the participants to the Form as imperfect or deficient instances of the essence that the Form represents/corresponds to. Eventually, and according to 'Predicationalist' approach, we allow that a Form F and its essence are in a sense identical, without regarding the Self-Predication statement itself as an identity claim (Code [3], Silverman [4] Ch.3). Rather, we suppose (as we analyze in the next section) that a Self-Predication claim asserts that there is a special primitive kind of ontological relation between a Form (subject) and its essence (predicate).

[1] The related phrase 'ἓν ἕκαστον εἶδος' in *Parm.* $132a1$ and its relation to the uniqueness thesis is analyzed by Cohen [5], pp.$433-466$. Analogous thesis is also present in other Platonic texts, e.g. *Phaedo* $100-101$ et al. .

[2] In *Phaedo* the Form is also referred as *the F itself* ($74a11-12$), which is the καθ' αὑτὸ F ($74b3-4$), 'the cause' (αἰτία) that 'makes' (ποιεῖ) things being F ($100c9-d8$), or the explanation of something being F.

Our mathematical model is based on the hypothesis (that is justified in the sequel) of the existence an infinite decreasing sequence consisting of sets of particulars predicated as F, according to their degree of participation (in increasing order) to F. If we assume that particulars in the same set have the same degree of participation F_i to F, then the sequence of sets S_{F_i} of particulars is denoted by $\{S_{F_i}\}_{i=0}^{\infty}$.

Finally, we claim that this infinite decreasing sequence $\{S_{F_i}\}_{i=0}^{\infty}$ converges to the Form-essence F, and in this context we construct a concrete topological framework in which the convergence takes place, and that in our view describes this process in an efficient manner. Henceforth, $\lim_{i\to+\infty} S_{F_i} = S_F \equiv F$, where we identify S_F with F since this set is singleton (by the uniqueness thesis). The convergence of the above sequence is understood as a mathematical one (in our topological framework) meaning that its greatest lower bound (g.l.b), namely F, is attained.

This terminating/limit F- Form should be also apprehended as compatible to the 'anupotheton arxēn' ('ἀνυπόθετον ἀρχὴν')[3] of *Republic's* language, but applied here for each particular predicate F. Moreover, this F-Form should be also considered as analogous to the 'final rung of Diotima's ladder' as presented in *Symposium* 210*eff*[4]. (In *Symp.* the above procedure is developed in the context of a particular Form, namely the Form of 'Beauty'.) Analogous procedure and the concept of the Form-essence F as a limit, which is the true essence F that the many particulars strive to approach and eventually come in touch-converge to it (essentially making a contact to it) is also presented in *Rep.*490*b*[5]. Similar procedures (as we shall see later) can be found through out the entire Platonic corpus, such as in *Republic*, *Theaetetus* 150*b*, 186*a*, *Sophist*, *Politicus*.

2 The Theory of Forms and the Foundation of the Argument

Before proceeding, we shall briefly present the historical background of the problem, as needed for developing and defending our arguments as well as the Main Claim.

We also note that we shall not deal with any issues related to the so called 'Imperfection Argument', as entailed primarily in *Rep.* $523-525$, or elsewhere in Platonic dialogues[6].

In the following, the schematic letter 'F' shall serve as a dummy predicate for any predicate for which there is a Form-essence.

The best guide to the separation of Forms is the claim that each Form is what it is in its own right, each is καθ' αὑτὸ being. In asking 'What is (the Form) F?', Plato seeks how and in which

[3] The term appears in *Rep.* $509b-511d$, in $510b$ and in $511b$, and shall be interpreted later in the paper. We do not give at the moment any translation, since any translation may lead to a specific interpretation. (In [6] *ad loc.* is translated as 'the principle that transcends assumption'.) For an extensive analysis of this passage and especially the concept and the status of this term we refer to Karasmanis [7], [8] and Benson [9].

[4] We shall see that this approach is compatible with fundamental mathematical concepts developed in Plato's academy, as well as with Plato's dialectic. For an elaborate and comprehensive exposition of Plato's dialectic see Robinson [10] Ch. 6, 7 and 10.

[5] We quote: *'...his passion would not be blunted nor would his desire fail till he came into touch with the nature of each thing in itself by that part of his soul to which it belongs to lay hold on that kind of reality, the part akin to it, namely, and through that approaching it, and consorting with reality really, he would beget intelligence and truth, attain to knowledge and truly live and grow, and so find surcease from his travail of soul ...'*.

[6] For example this argument does not posit a Form even for every property-name; it posits a Form for the predicate *large* but not a Form for the predicate *man*. And it supports that we can infer that there is a Form of F, only when we have a group that consists of imperfectly F things. Namely, the imperfection argument posits Forms both for restricted range of predicates and also a restricted range of groups. (For further details on the 'Imperfection Argument' we refer to G. Fine [1].)

manner a Form F is independent from any of its material instances, and in some sense independent of anything else. Eventually, he concludes that each Form is in its own right, it is independent, in virtue of its essence-οὐσία.

There are three main approaches to the theory of Forms. A fundamental notion that we clarify at first is the one of 'Self-Predication' (not to be confused with 'Self-Participation') and how it is comprehended when applied to Forms.

Self-Predication

'The Form corresponding to a given character itself has that character.'

The connection between the Form and the essence being predicated of is exhibited in the *Republic's* formula (477ff) that a given being-ὄν is completely or perfectly F.

The debate over 'Self-Predication' involves both statements and what the statements are about, i.e., the ontological correlates of these statements. (Thus, at times it may be important to distinguish linguistic predication from ontological predication.) In investigating self-predication statements we consider, without loss of generality, a particular example: 'The Just is just'. Perhaps it is easiest to distinguish three factors: The subject or subject term, e.g. 'The Just', the linking verb, 'is', and the predicate adjective 'just'. Apparently both the subject and the predicate adjective, 'The Just' and 'just', refer to the same thing, namely the Form of Justice (similarly with other predicates e.g. 'The Beauty is beautiful'). One question then concerns the copula, or linking verb: in what manner is the predicate related to the subject, or how is the Form related to itself?

Vlastos [11] in his seminal discussion of 'Self-Predication' maintained that we should understand the relation between the Form and itself to be the same as that between a particular and the Form. This is to say that 'Justice is just' in the same way as 'Socrates is just', or that 'Beauty is beautiful' in the same way as 'Helen is beautiful', or that the 'Circle Itself is circular' in the same way as a ball; both are round. Let us label in this way of understanding the copula in self-predication statements as 'characterization'. According to this view then, Beauty 'Is' a beautiful thing, an item to be included in an inventory of beautiful things right along with Helen.

Some scholars, e.g., J. Malcolm [12], while accepting this characterizing reading of the 'Is', deny that the property predicated of the Form and the particular are exactly the same. According to this approach, namely the Approximationist one, the Form is considered as the perfect instance of the property it stands for. A particular that participates in the Form is an imperfect or deficient instance, namely it has a property that approximates the perfect nature of the Form. For instance, the Circle itself is perfectly circular. A drawn circle, or a round ball, is deficient in that it is not perfectly circular, not exactly 360 degrees in circumference (or its area is not exactly π). It follows that the very properties particulars possess will differ from the property 'of the same name' possessed by the Form. Note that while the appeal to the perfection of the mathematical properties is great, even in these cases it is doubtful that Plato adopts an Approximationist strategy (see Nehamas [13], [14]).

In relation to this, it is worth mentioning a contemporary analogue due to the linguist Noam Chomsky. Chomsky describes what he calls 'The Argument from the Impoverished Stimulus' as a classic rationalist argument. It notes that we classify physical shapes that we experience (written, printed, drawn, et.c.) as inexact representations of geometrically perfect regular figures (squares, circles, triangles, et.c.). Why do not we classify them as exact representations of irregular figures? The idea is that our sensory stimuli are 'impoverished.' We never experience perfect squares, circles, triangles, et.c. Yet we have these concepts, we endow them with a mathematical definition based on an 'axiomatic system', and we classify things accordingly. How did we acquire these concepts if we have never experienced anything that they (literally) apply to? (For further discussion on this, see Cohen [15]).

There is another approach, namely the 'Predicationalist' one, see Nehamas [14], Code [3], Silverman [4], that is denying that self-predication statements signal that the Form is characterized by the property it constitutes, and while ultimately it allows that a Form and its essence are identical, it does not regard the self-predication statement itself as an identity claim (see Code[3], Silverman [4] Ch.3). Rather, a self-predication claim asserts that there is a special primitive kind of ontological relation between a Form (subject) and its essence (predicate). According to the predicationalist reading, the relation connecting an essence with that Form of which it is the essence of, is 'Being' (see Code [3], $425 - 9$). Henceforth, this approach begins from the two relations of 'Partaking' or 'Participation' (methexis-μέθεξις) and 'Being', introduced in the last argument of the dialogue *Phaedo* ($100a - 107a$).

In the first place we treat participation as a relation between material particulars and Forms, the result of which is that the particular is characterized by the Form of which it partakes-participates. So, Helen, by partaking/participating to the Form of Beauty, is characterized by beauty; Helen, in virtue of partaking, is (or, as we might say, becomes) beautiful.

In general, any particular is characterized by the Form in which it participates, and whatever each is, it is by participating in the appropriate Form. On this account, then, there can be Forms for each and every property had by particulars (*Phaedo* $100 - 101$, esp. $100c6$). In contrast to the characterizing relation of 'Partaking', the relation of 'Being' is always non-characterizing. Each Form F, is its essence (ousia-οὐσία), which is to say that the relation of 'Being' links the essence, e.g. of beauty, to the subject, e.g. Beauty Itself.

'Being', then, is a primitive ontological relation designed exclusively to capture the special tie between that which possesses an essence, and the essence possessed. Put differently, whenever essence is predicated of something, the relation of 'Being' is at work. We note that by 'primitive' we do not mean to suggest that Plato does not study (what) 'Being' (is). Nor do we mean to suggest that everything else in the metaphysics can somehow be deduced from it. Rather, we mean to indicate that the relation of Being is not explained by appeal to another more basic relation or principle. This particular property of 'Being' renders the Form-essence, as we shall see later, analogous to the anupotheton arxēn of *Republic*, establishing the Form as independent and in a sense self-explanatory.

In light of the above, following Predicationalist approach, we emphasize that the copula 'Is' is used to represent the predication relation of Being, e.g., 'Justice Is just'. The predicate in such self-predication statements stands for the (real) definition of the Form, what it is to be F (see Nehamas [13], [14]). That is, 'Justice Is just' is short for 'Justice is what it is to be just'. Henceforth, each Form Is its essence via the predication relation of Being, which can be captured in:

I. Each essence is the essence of exactly one Form.

II. Each Form has (or is) exactly one essence.

The above properties capture the ontological force of the expression that each Form is 'monoeides': of one essence. Furthermore, they express the existence of an 'one to one' correspondence between the set of Forms and the set of Essences. This 'one to one' correspondence is declared by the linking verb 'Is'. In light of these principles, and in keeping with the account of the ontological relation of 'Being', it follows that each Form self-predicates, in so far as each Form 'Is' its essence.

Henceforth, since every Form must be its respective essence, self-predication is a constitutional principle of the very theory of Forms. Indeed, since the only thing a Form 'Is', is its essence, each Form is 'monoeides', 'of one essence'[7].

In virtue of 'Being' its essence, each Form Is something regardless of whether any particular does or even may participate in it. Thus, each Form is separate from every particular instance of it. Moreover, since its essence is predicated of the Form independently from our knowledge of the Form or from its relation to another Form, a Form is not dependent on anything else. On this definitional interpretation of separation, an item is separate just in case the definition (essence) is predicable of it and not of what it is alleged to be separate from. Therefore, a Form is separate from particulars that partake to it or any particular, if the essence is predicable of the Form and not predicable of the particular/s. Whether or not a Form is existentially separate, i.e., whether it exists separate from everything else, turns on whether one thinks that being an essence qualifies

[7] Thus,'compresence' does not characterize Forms. For this see G. Fine [1].

the Form as existing. To the extent that Plato recognizes the notion of existence, since being an essence seems, by Plato's lights, to be the superlative way to be, it is likely that Forms are both definitionally and existentially separate.

3 The Main Claim of the Article

Our analysis is based on both Approximationist and Predicationalist approaches, without assuming that all Forms are characterized by 'Self-Predication' in the strict sense of Vlastos [11], but rather according to the predicationalist approach[8] (see also the previous paragraph).

We assume that particulars are depended on Forms, whereas Forms, as definitionally and existentially separate, are not depended on them. Particulars strive to be such as the Forms are, and thus in comparison to Forms are imperfect or deficient.

According to the above and the arguments from the previous section we adopt the following system of Hypotheses/Definitions that shall be used and assumed throughout the paper (justification shall be provided within this article based on Plato's work). We also denote the set of natural numbers including zero as $\mathbb{N}$, that is $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

System of Hypotheses/Definitions

1. Each essence is the essence of exactly one Form.

2. Each Form Is (or possesses) exactly one essence; each Form is 'monoeides'.

3. Each Form F is 'identified' with its essence F-ness. This identification represents a special ontological relation-predication between the Form F and its essence. Namely, it is this sole essence in the highest (ultimate) degree.

4. For any particular p, and for any property-essence F-ness, if F-ness is predicated (or predicatable) of p, then p has F-ness.

5. If a particular p has F-ness, then p has a degree of the essence F-ness, but not the ultimate degree.

6. Different particulars that are predicated as F may have different degrees of participation F_i, $i \in \mathbb{N}$, to the essence F-ness.

7. The collection of all the particulars predicated as F having the same degree of participation F_i, $i \in \mathbb{N}$, to the Form-essence F, form a set S_{F_i}.

According to hypotheses $6, 7$, if $p \in S_{F_i}$, then p has degree of participation F_i to the Form-essence F.

8. If a particular has a degree of participation F_i, $i \in \mathbb{N}$, then we assume that it also has all the lower lever degrees of participation. That is, $\forall i, j \in \mathbb{N}$:

(i) if $i < j$, then $F_i < F_j$;

(ii) if $i < j$ and $p \in S_{F_j}$, then $p \in S_{F_i}$. Thus, $S_{F_j} \subset S_{F_i}$, and hence the sequence $\{S_{F_i}\}_{i=0}^{\infty}$ is decreasing.

9. Since the Form-essence F is 'monoeides' $(1, 2)$, then the set S_F containing F is a singleton, thus we identify it with F. That is $S_F \equiv F$ and we write F instead of S_F.

The hypothesis 3 essentially describes the property of 'Self-Predication' (see also previous section). That is, 'Self-Predication' tell us that the essence of F, namely F-ness, is predicated as being F, via the predication relation of 'Being'. Which, as stated in the previous section, it is

[8] For example, the Circle Itself-the Form 'Circle'-is perfectly circular. Whereas, a drawn circle, or a round ball, is deficient in that it is not perfectly circular. In contrast to this, if for example we consider the Form of 'Multiplicity', this Form cannot have the property of being multiple (more than one) because this contradicts the uniqueness 'monoeides' of the Form. Rather, we accept that the Form of 'Multiplicity' it captures perfectly the essence of being multiple. For this, recall N. Chomski's 'Argument from the Impoverished Stimulus' stated in the previous section.

an ontological relation that captures the special tie between that which possesses an essence and the essence possessed.

We adopt the position that in case of Forms, in contrast with that of the particulars predicated as F, the Form F is identified with its unique essence (hypotheses $1, 2, 3$). We also note that the hypotheses 4 an 5 clearly assert that particulars are what they are in virtue of the Form being what it 'Is'.

Our analysis and interpretation points (as we will support analytically in the next section) to an existence of an infinite increasing sequence $\{F_i\}_{i=0}^{\infty}$ of degrees of participation-methexis of the particular predicated as F to the Form F and hence to the essence that the Form is identified to. This is expressed clearly in hypotheses $3-8$. This is also compatible with Plato's theory of degrees (anavathmoi) of participation/partaking (methexis). The larger the value of i, the higher the degree of participation of the particular/s predicated as F to the essence F-ness (see hypotheses $6, 8$). Dually, the Form-essence F has an analogous degree of presence-parousia to any particular in S_{F_i} (depending of the value of i).

Now consider (as in hypothesis 8) the infinite decreasing sequence $\{S_{F_i}\}_{i=0}^{\infty}$ formed by the sets of particulars predicated as F with degrees of participation F_i, $i \in \mathbb{N}$, to F:

$$S_{F_0} \supset S_{F_1} \supset S_{F_2} \supset S_{F_3} \supset ... \supset S_{F_k} \supset S_{F_{k+1}} \supset \dots \quad , k \in \mathbb{N}. \qquad (\star)$$

In the above, S_{F_0} denotes the set of particulars of minimal degree of participation to the Form-essence F, in order for them to be predicated as F. For example, in the case of the Form of Beauty, F_0 denotes the least degree of Beauty in order for a particular to be called 'beautiful' and hence to participate in the Form of Beauty and thus being a member of S_{F_0}.

A central thesis in our Main Claim is that the sequence in $(\star)$ eventually converges (under a certain topology) to the unique Form-essence $F \equiv S_F$.

In section 5 we establish and analyze the mathematical/topological framework that the above procedure takes place.

The main result of this article is presented in the following claim. The justification and support of this claim is mainly presented in the sequel, where we argue that this approach is consistent with Plato's philosophy and his dialectic theory.

Main Claim:

(a) *For every Form-essence F there exists an infinite decreasing sequence $\{S_{F_i}\}_{i=0}^{\infty}$ of sets of particulars predicated as F (equation $(\star)$), with corresponding degrees of participation that form an increasing sequence $\{F_i\}_{i=0}^{\infty}$.*

(b) *The infinite decreasing sequence $\{S_{F_i}\}_{i=0}^{\infty}$ from part (a) converges to the unique F-Form (as a limit) under a certain topology $\mathcal{T}$. That is $\lim_{i \to +\infty} S_{F_i} = F$.*

(c) *The presence-parousia of the essence-Form F to each of the particulars in the sets S_{F_i}, having degree of participation F_i, is described by the dual topology $\mathcal{L}$* of $\mathcal{T}$, namely $\mathcal{L} = \mathcal{T}^*$

In Plato's language this (limiting) F-Form from part (b) is refereed also as the unique (according to the 'uniqueness thesis') 'F-itself', 'καθ᾽ αὑτὸ' F (see *Phaedo* $74b3-4$, $100b5-7$ et al.). In the sequel, especially in the next section, we argue that the 'greatest lower bound' of the sequence $(\star)$ is attained (as being a mathematical limit) and this is exactly F.

To be consistent with Plato's academy mathematics we should mention that the mathematical concepts of 'apeiron', 'peras', 'limit', 'density' as well as the one of 'convergence' were discussed and comprehended in some extend in the academy. The author analyzed the Platonic status of these concepts in Chailos [16], Sec.4, based primarily on *Philebus* and *Theaetetus*, as well as some Neo-Platonic theory[9]. There, it was presented an elaborate treatment of the notions of 'countable' and 'uncountable' infinities, in relation to Eudoxus' method of exhaustion for approximating lengths

[9]For an extensive presentation of Mathematics in Plato's Academy consult Fowler [21], pp.322-328, Anapolitanos [18] and Negrepontis [20].

and areas, that is very close to the notion of what we call now 'Dedekind cuts' (see also Taylor [17], ch.20, 'Forms and Numbers', Anapolitanos [18], Karasmanis [19], Negrepontis [20]).

Nevertheless 'X.2 in Anonyma Scholia to Euclid' contains what might be considered the Platonic criticism to general 'Dedekind cuts' introduced by Eudoxos. In spite of this, Plato had a good grasp on the concept of convergence and infinity. In particular he had used the technique of 'anthyphairesis' (ἀνθυφαίρεσις) throughout his work to establish the true nature of his Forms-essences. More specifically, in his later dialogues, he started from the method of 'Division and Collection' to pass to the study of 'the commensurable in power pairs of lines' magnitudes, that have infinite but palindromically periodic anthyphairesis, analogous to the 'square root of n, when n is a non square number'[10].

4 The Defense of the Main Claim and Related Issues

4.1

Here we establish the part (a) of the main claim by providing evidence that Plato in his works support this thesis and he is consistent to it.

The approximative nature of the essence of F by particulars predicated as F is found in the dialectical kernel of Diotima's lecture on the love of Beauty in *Symposium* $209e5 - 211d1$. Diotima's lecture is a vivid description of a hierarchical progressive model for acquiring knowledge and eventually make a tangential contact to the Form F, which is 'identified' to the essence F-ness. The whole procedure is done through successive approximating levels of degrees (anavathmoi) of participation, or considered predicationally, analogous degrees of the true judgement of a particular being predicated as F to the Form-essence. The procedure is crowned with the sudden (ἐξαίφνης) appearance of the Form. Similar to this approach is present in many Platonic texts (as we shall se later), e.g. *Republic* $490b$ (see also note 5) and is a fundamental one within the Platonic dialectic (see also note 11).

More specifically, Plato in *Symp.* $210e$, among others, states *'...passing from view to view of beautiful things, in the right and regular ascent,..'*, noting also that the ascension to the final rung-degree, corresponding to the ultimate Form (of 'Beauty') itself, has to be done in a *'correct and orderly succession'* (ἐφεξῆς ὀρθῶς τὰ καλά). This is even more clear in *Symp.* $211b - c$ where the nature of the ascending procedure to the Form of Beauty is analyzed. From this passage we hold on the phrase 'ὥσπερ ἐπαναβασμοῖς χρώμενον'-'as on the rungs (anavathmoi) of a ladder'-to state that the procedure is done through a successive (ἐφεξῆς) approximating increasing ('ἐπανιών'-'that ascends') levels of degrees of participation/predication (see hypothesis 8). According to Plato this describes the 'right approach' (ὀρθῶς) for 'almost being able to lay hold of the final true F-Form' ('σχεδὸν ἄν τι ἅπτοιτο τοῦ τέλους'), which constitutes also the ultimate goal and the conclusion of the whole procedure. According to Vlastos [22] the whole procedure moves 'closer step by step to the Beauty itself'. We have to state that nothing prevents us from assuming that the same model holds analogously for all Forms (essences) and is not restricted only to the Form of 'Beauty' (similar procedure, as we shall present, is followed in Plato's *Republic, Theaet., Soph. et al.*.)

The fundamental role of the above procedure (as in Main Claim) in Plato's dialectic, is clearly presented in *Rep.* $509d - 511e$, as well as in $532d - 535a$[11]. Plato in the analogy of 'Simile Divided

[10] Negrepontis in a series of papers, [24],[25],[20], provides an original and extensive analysis of the technique of anthypharesis and its status in the dialogues *Parmenides, Theaetetus, Sophistes, Politicus*. He argues that this technique is essential to establish the ontological and epistemological status of the Platonic Beings-Forms. He proves that Platonic Beings are characterized by palindromically periodic anthyphairesis.

[11] In relation to this we mention in *Rep.*$511b$ the phrase 'οἷον ἐπιβιβάσεις'. For an analysis of these passages from *Republic* and their relation to Plato's dialectic consult Karasmanis [7], [8], Benson

Line', *Rep.* $509d - 513a$, advances this hierarchical progressing model of levels, that leads to the anupotheton arxēn, claiming that the whole procedure is done in the realm of noēsis (νόησις) and precisely in the section of epistēmē (ἐπιστήμη), via the exercise of dialectic method.

Analogous arguments are advanced in the seminal work of Proclus' 'Commentary on Plato's Parmenides' [23] (particularly in $879.15 - 28$, $881.23 - 33$)[12].

The previous analysis, together with the analysis of the next subsection, strongly supports and points to the existence of a model based on an hierarchy among the plurality of degrees F_i, $i \in \mathbb{N}$, of participation-methexis of the particulars to the Form-essence F. In this procedure, the larger the value of i, the higher the degree of participation of the corresponding particular to the Form F is. Henceforth, the sequence $\{F_i\}_{i=0}^{\infty}$ is increasing (this supports hypothesis 8(i)). Consequentially, and according to previous section (hypoyheses $6, 7$) the sequence $\{S_{F_i}\}_{i=0}^{\infty}$ in $(\star)$ is necessarily decreasing (this supports hypothesis 8(ii)).

Considering all of the above, we conclude that the part (a) of the Main Claim is established.

4.2

In this section we aim to establish the second part of the Main Claim, without discussing issues related to the specific topology in play. The description of this topology is presented and analyzed in the next section.

This is done by claiming that the decreasing infinite sequence $\{S_{F_i}\}_{i=0}^{\infty}$ is convergent. Henceforth, its 'greatest lower bound' is attained and it is the limit of this sequence. This limit could be considered as the sole essence F-ness, the F-itself (καθ' αὑτὸ 'F'), for the predicate in concern. It is the necessary and sufficient condition in order for the particulars-participants to be predicated as F, according to their degrees of participation in $\{F_i\}_{i=0}^{\infty}$.

Furthermore, we address the various questions that arise regarding the nature of the limit-Form F and the way it should be understood in relation to the various degrees F_i, $i \in \mathbb{N}$, of participation of the particulars predicated F to it. Of course this has to do with the topological framework of the problem (see Sec. 5).

The limit-Form F is the unique Form, as defined in *Phaedo*, the 'F itself ' ($74a11 - 12$), the 'F without qualification' ($74b$*ff*), the 'F' that it can never seem non-'F'($74c1 - 3$) (see also note 2). In *Symp.* $211c$ this F is further understood as the unchangeable end, the goal, the conclusion ('αὐτὸ τελευτῶν ὃ ἔστι') of the convergent ascending procedure of degrees of participation $\{F_i\}_{i=0}^{\infty}$ to it, identified with the essence F-ness. Similar terminology and way of apprehending this 'F-itself' is encountered in many Platonic dialogues, such as *Phaedo* et al.. For example in *Phaedo* $101e$ it is described as the termination of such procedure to the 'One' Form-F which is the 'adequate' ('ἕως ἐπί τι ἱκανὸν ἔλθοις') existential and explanatory cause for particulars to be predicated as F. Analogous procedure is also developed elaborately in an abstract manner in *Rep.* $509e - 511d$ and shall be discussed in the sequel. (See also Karasmanis [7] for the analysis of hypothetical method in this passage.)

As we have already mentioned, the dialectical method-procedure possesses a central thesis in Platonic theory and runs throughout the Platonic corpus. It is meticulously presented and analyzed in *Rep.* $509c{-}511e$ and $532d{-}535a$. This dialectic procedure could be extended beyond the *Republic* case and could be applied for each particular F- Form/essence.

In *Rep.* $510b$ Plato states clearly that the highest rung of the ladder is not reached until the entire domain of epistēmē has been exhausted via the dialectic process, under the cause of noēsis. This highest rung is referred as anupotheton arxēn (ἀνυπόθετον ἀρχήν), which is established by an exhaustive scrutiny, being higher than all premises-hypotheses ('ὑποθέσεις'). It is higher, in

[9], J. Annas [26] ch 10, 11, as well as Robinson [10] ch. 6, 7, 10.

[12] We quote: '*...And from there in turn he will be chasing after unities of unity, and his problems will extend to infinity, until, coming up against the very boundaries of intellect, he will behold in them the distinctive creation of the Forms, in the self-created, the supremely simple, the eternal...*'.

the sense that contrary to them has an axiomatic status (playing the role of a system of axioms), it is a 'non-hypothetical' one, situated in the highest point of the intelligible world (in *Republic's* $509d - 513a$ 'Simile Divided Line') and does not require derivation. Rather, it provides an account and has an explanatory role (quasi-proof) through its presence in the particulars/elements of the sets S_{F_i}, $i \in \mathbb{N}$, (see also Karasmanis [7], [8] and Benson [9], p.190).

Furthermore, this anupotheton arxēn should be comprehended not as a transcendental ontological mystery but in the mathematical sense of the attainable 'greatest lower bound', g.l.b, of the decreasing sequence $\{S_{F_i}\}_{i=0}^{\infty}$. Henceforth, it is identified with the sole essence F-ness, via the primitive ontological relation of 'Being' as developed in the previous section (see ' System of Hypotheses/Definitons'). The procedure expressed by $\{S_{F_i}\}_{i=0}^{\infty}$, using Plato's terminology, could be viewed as the one of 'becoming'-gignesthai ('γίγνεσϑαι') that leads to the 'unconditional', 'immutable', 'objective', 'unchangeable', 'perfect' and 'unique' 'Is'-'εἶναι' [13]. This 'Is', as characterized above, is the 'monoeides' essence of F-ness, identified with the true Form F.

Apart from *Republic*, it should be apprehended as the 'One' ('μονοειδὲς' in *Phaedo's* language) that should be parallelized with the highest-terminating rung of Diotima's ladder, which is tangent to the very essence of F-ness[14]. In *Philebus* this approach is even more clear. At $16c$ it is stated that a Platonic Form is the 'Mixture' ('Meikton') of two principles/classes, thsese of 'Infinite' ('Apeiron') and 'Finite' ('Peras'). Here we could argue that 'Peras' is the limiting point of the ascending procedure expressed by $\{F_i\}_{i=0}^{\infty}$; thus, we could view it as expressing the Form-essence F. Dually, we could conceptualize it as the attainable g.l.b of $\{S_{F_i}\}_{i=0}^{\infty}$. In *Philebus* ($23d6 - 7$, $26d7 - 9$, $27d6 - d10$, et al.) the class of 'Mixed'-'Meikton' is formed by imposing, with the aid of the class of 'Finite'-'Peras', a limit, a due measure on the class of 'Infinite'-'Apeiron' via a specific process that leads to the unchangeable perfect 'Is' (εἶναι). This process, characterized in $26d8$ as 'genesis eis ousian'-'γένεσις εἰς οὐσίαν', in $55a3$ is comprehended as the becoming, the generation-gignestai[15], the limiting procedure that converges to the stable 'Being', the immutable reality, the unchangeable 'Is'. Analogous to the above interpretation is advanced also in Plato's *Tim.* $29c4$, cf. *Sophist* $232c7 - 9$.

Furthermore, and in the line of thought of *Philebus*, Proclus in 'Commentary of the First Book of Euclid's Elements' [27] is arguing, using the concepts of 'Peras' and 'Apeiron', in order to apprehend and establish the convergence via the notion of the limit. Moreover, in his work 'Commentary on Plato's Parmenides' [23] $881, 23 - 33$[16] analyzes the concept of infinite ('apeiron') process that arrives to a terminating mental point- 'noeron peras', via the process of the intellect noēsis. The term 'noeron peras' should be apprehended as analogous to the one of 'anupotheton arxēn'. The whole procedure is analogous to this of 'Simile Divided Line' (*Rep.*$509d - 513a$) and Plato's dialectic theory (see note 11).

These concepts in *Philebus*, as we mentioned earlier, were discussed, studied and analyzed in detail by various authors (see Negrepontis[20], Karasmanis[19], Chailos[16]).

[13]In *Symp.* $211a$ Plato characterizes this 'Is' as: '*... ever-existent and neither comes to be nor perishes, neither waxes (growths) nor wanes (declines, decreased)...*'. In $211b$ is characterized as unchangeable, affected by nothing. Furthermore, in $211c$ this 'Is' is revealed at the end of the ascending procedure, characterized as the very essence of the F-ness.

[14]see *Symp.* $211b - c$ where that F is the Form of 'Beauty', as well as the previous subsection 4.1.

[15]The word 'genesis' (conceptualized as generation) appears often in Plato and is commonly presented as the opposite of 'destruction'-'φϑορά'.

[16]We quote:'*...And from there in turn he will see other more comprehensive unities, and he will be chasing after unities of unity, and his problems will extend to infinity, until, coming up against the very boundaries of the intellect, he will behold in them the distinctive creation of Forms, in the self-created, the supremely simple, the eternal...*'. A similar line of thought is present also in Plotinus, *Enneads* $2.4.15, 15 - 16$.

The limiting procedure discussed in this section, as well as the notion of the Forms-essence as limits of such procedures, are also present and are of importance in many other Platonic texts (of the middle and late period), such as *Symposium*, *Phaedrus*, *Philebus*, *Sophist*, *Politicus*, *Epistle* 7 *et al.*.

For example, recall that Plato in *Symp.* $210e - 212a$ characterizes the limit Form-essence F (here is the Form of 'Beauty') as the termination of a limiting procedure, performed in the most perfect manner via degrees-rungs of participation, that is revealed 'abruptly, suddenly'-'ἐξαίφνης'. Additionally, he states that this Form exist unconditionally and is 'the perfect thing, the wondrous and beautiful in nature'-'θαυμαστὸν τὴν φύσιν καλόν' [17]. In $211e$ he characterizes it as 'the divine beauty itself, in its unique form'- 'θεῖον καλὸν μονοειδές'[18], and in *Symp.* $212a$ as the 'tangent-contact to the truth'-'τοῦ ἀληθοῦς ἐφαπτομένῳ'. In *Symp.* $211c9$ is called the 'terminating point of the ascending procedure'- 'αὐτὸ τελευτῶν ὃ ἔστι'[19]. Analogous procedure is the one of *Rep.* $490b$ (see note 5). Lastly, in Plato's *Epistle* $7.341c$ we encounter similar terminology which emphasizes that the true-Form is revealed suddenly as the culmination/ending of an ascending in degree of partaking/methexis procedure[20].

5 The Mathematical Framework-Projective and Inductive topologies

In this section we complete the proof of the part (b) and establish the part (c) of our Main Claim. For this, we present the necessary mathematical framework for our Main Claim. We primarily focus on the description of the topologies in play, that as we claim provide an efficient and solid interpretation model for the description of the dual processes of:

1. The participation/partaking (methexis) of the many particulars predicated as F to the Form-essence F, according to their degree of participation to it.

2. The presence (parousia) of the Form-essence F to the particulars predicated as F , in analogy to their degree of participation to F as in 1.

Recall that $\{F_i\}_{i=0}^{\infty}$ denotes the infinite increasing sequence of degrees of participation of the particulars predicated as F to the Form F, and hence to the essence F-ness. In addition, for each $i \in \mathbb{N}$, we consider the set S_{F_i} of (the many) particulars that have a common degree of participation F_i to the form-essence F, and we endow this set with a certain topology $\mathcal{T}_i$. Now, since S_{F_i}, $i \in \mathbb{N}$, are discrete sets, we could consider $\mathcal{T}_i$ to be the Discrete Topology on S_{F_i}; that is, the topology induced by the discrete metric. It is of crucial importance that $(S_{F_i}, \mathcal{T}_i)$ becomes (with this particular topology) a complete metric space. Furthermore, we construct the projective limit associated with S_{F_i} and we endow it with the projective topology[21]. Also recall from section 3, hypothesis 9, that S_F is identified with F; $S_F \equiv F$.

Now, for all $i \leq j$, $i, j \in \mathbb{N}$, define the connecting maps

$$\mu_{ij} : S_{F_j} \to S_{F_i},\ i \leq j, \text{ where } \mu_{ij}(f_j) = f_i \text{ is the imbedding of } f_j \in S_{F_j} \text{ to } S_{F_i}. \qquad (5.1)$$

[17] A similar terminology is used in *Phaedrus* $250b$.

[18] In *Republic* $398a$ this perfect Form is characterized as 'divine and holy'-'ἱερὸν καὶ θαυμαστόν'.

[19] For an analysis of the passage $201d - 212c$ in *Symp.* see Taylor [17], Ch 9, Sec. 8 and Vlastos [22].

[20] *Epistle* $7341c$ '*...but, as a result of continued application to the subject itself and communion therewith, it is brought to birth in the soul on a sudden, as light...*'. Analogous approach is encountered also in Plotinus *Enneads* 43.17.

[21] Projective limits and projective topologies, as well as their dual notions, are studied in Shaefer [28] ch $2-4$, and an application of them in a particular Analysis problem can be found in Chailos [29].

Note that for all $i, j, k \in \mathbb{N}$, $i \leq j \leq k$, holds $\mu_{ik} = \mu_{ij} \circ \mu_{jk}$. Now let S_{F_∞} to be the subspace of $\prod_{i=0}^{\infty} S_{F_i}$ whose elements $f = (f_1, f_2, \ldots)$ satisfy the relation $f_i = \mu_{ij}(f_j)$ for all $i \leq j$. S_{F_∞} is called the 'projective limit' of the family $\{S_{F_i}\}_{i=0}^{\infty}$ with respect to the mappings μ_{ij} and is denoted by $S_{F_\infty} = \varprojlim \mu_{ij}(S_{F_i}, \mathcal{T}_i)$.

We consider the projection map p_i of $\prod_{i=0}^{\infty} S_{F_i}$ onto S_{F_i}, $i \in \mathbb{N}$, and we set μ_i to be its restriction to S_{F_∞}. Additionally, we endow S_{F_∞} with the projective topology $\mathcal{T}_\infty$ with respect to the family $\{(S_{F_i}, \mathcal{T}_i), \mu_i\}_{i=0}^{\infty}$. That is the coarsest topology on S_{F_∞} for which each of the mappings $\mu_i : S_{F_\infty} \to (S_{F_i}, \mathcal{T}_i)$, $i \in \mathbb{N}$, is continuous.

An element $f_i \in S_{F_i}$ is called a 'representative' of $f \in S_{F_\infty}$, if $\mu_i(f) = f_i$, $i \in \mathbb{N}$. Note that each element of S_{F_∞} has a unique representative in each S_{F_i}, but that an element of S_{F_i} does not necessarily represent a unique element of S_{F_∞}. Furthermore, note that there is no restriction of generality in assuming that a projective limit is reduced, in the sense that for each $i \in \mathbb{N}$, the projection $\mu_i(S_{F_\infty})$ is dense in S_{F_i}. (See Schaefer [28], Ch. IV, Sec. 5.)

In support of our model, it is important to note that a philosophical analogue of this projection is found in Proclus [23] $878, 1-890, 38$. According to Proclus' thesis, such an approach is compatible with 'presence-parousia' of Forms in their participants, since the creative role of Forms is preserved, without assuming 'likeness' or 'divisibility' of the Form F to its participants predicated as F.

The reason of choosing this particular topology shall be made even more clear in the sequel where we investigate its dual topology, namely the inductive topology.

The following is the definition of the exhaustion space.

Definition 5.1. *We say that a function f belongs to the class $(S_{F_\infty}, \mathcal{T}_\infty)$, if $f \in (S_{F_i}, \mathcal{T}_i)$, for each $i \in \mathbb{N}$, where $\mathcal{T}_\infty$ is the projective topology with respect to the family $\{(S_{F_i}, \mathcal{T}_i), \mu_i\}_{i=0}^{\infty}$. We call $(S_{F_\infty}, \mathcal{T}_\infty)$ the exhaustion space of the family $\{(S_{F_i}, \mathcal{T}_i), \mu_i\}_i$, $i \in \mathbb{N}$.*

In order to capture mathematically the duality of the processes $1, 2$ (stated earlier), we need to investigate the topological dual of $(S_{F_\infty}, \mathcal{T}_\infty)$. For this purpose we need the notion of inductive topologies, since as we shall see in theorem 5.2 there is a certain duality between inductive and projective topologies.

For $i \in \mathbb{N}$, set Y_i to be the algebraic dual of S_{F_i}, that is the space of linear functionals on S_{F_i}. The notion of duality is needed, since the concept of the 'presence-parousia' of F to the elements of S_{F_i} can be viewed as the dual concept of the one of 'participation-methexis' of elements in S_{F_i} to F (with a certain degree of participation). Therefore, the concept of the 'presence-parousia' of F to elements of S_{F_i} can be comprehended mathematically as a linear functional on the topological space $(S_{F_i}, \mathcal{T}_i)$, $i \in \mathbb{N}$.

Furthermore, suppose that each Y_i has a topology $\mathcal{L}_i$. For all $i, j \in \mathbb{N}$ set ϕ_{ij} to be the dual maps of μ_{ij}. That is, $\phi_{ij} = \mu_{ij}^*$ are defined as:

$$\phi_{ij} : Y_i \to Y_j,\ i \leq j, \tag{5.2}$$

where if y_i, y_j are elements of Y_i, Y_j respectively, then $y_j = \phi_{ij}(y_i) = y_i \circ \mu_{ij}$. This and (5.1) imply that for $f_i \in S_{F_i}$, $f_j \in S_{F_j}$, $i \leq j$, the action of y_j on f_j is identified with the action of y_i on f_i.

Now we define the natural injections $g_i : Y_i \hookrightarrow \bigoplus_{i=0}^{\infty} Y_i$ and we let K denote the (closed) subspace of $\bigoplus_{i=0}^{\infty} Y_i$ generated by the closure of the ranges of the linear maps $g_i - g_j \circ \phi_{ij}$ of Y_i into $\bigoplus_i Y_i$, where $i, j \in \mathbb{N}$ run through all pairs such that $i \leq j$. Let also $p : \bigoplus\limits_i Y_i \to (\bigoplus\limits_i Y_i)/K$. The quotient space $Y^\infty \equiv (\bigoplus_i Y_i)/K$ is called the inductive limit of the family $\{Y_i\}_{i=0}^{\infty}$, with respect to the mappings ϕ_{ij} and is denoted by

$$Y^\infty = \varinjlim \phi_{ij}(Y_i, \mathcal{L}_i).$$

For all $i \in \mathbb{N}$ set ϕ_i to be the restriction to Y_i of the map $p : \bigoplus_i Y_i \to (\bigoplus_i Y_i)/K$ (that is, the imbedding of Y_i into Y^∞). Now provide Y^∞ with the inductive topology $\mathcal{L}^\infty$ with respect to the family $\{(Y_i, \mathcal{L}_i), \phi_i\}_{i=0}^{\infty}$. This is the finest locally convex topology that makes each of the mappings $\phi_i : (Y_i, \mathcal{L}_i) \to Y^\infty$, $i \in \mathbb{N}$, continuous and Y^∞ complete (see Schaefer [28] Ch. 2.6). This particular topology captures efficiently the dual processes 1, 2, as stated at the beginning of this section. One of the primary reasons is that this topology on Y^∞ is the finest one (as the hull topology), among all the topologies that make each of $\phi_i : (Y_i, \mathcal{L}_i) \to Y^\infty$ continuous. Hence, it is the best possible refinement that makes the linear functionals on $(S_{F_i}, \mathcal{T}_i)$, $i \in \mathbb{N}$, continuous.

An element $y_i \in Y_i$ is called a 'representative' of $y \in Y^\infty$, if $\phi_i(y_i) = y$. Note that for $i \in \mathbb{N}$, each element of Y_i represents at most one element in Y^∞, but that an element of Y^∞ does not necessarily have a unique representative in Y_i. Furthermore, it is clear that a given $y \in Y^\infty$ need not have a representative in each Y_i, $i \in \mathbb{N}$. Set I to be the subset of $\mathbb{N}$, such that $y \in Y^\infty$ has at least one representative in each element of $\{Y_i\}_{i \in I}$.

In order to prove that there is a certain duality between inductive and projective topologies (as constructed above) we shall need the Mackey-Arens theorem that characterizes the locally convex topologies consistent with a given duality (Schaefer [28]).

Suppose that X is a vector space over a field K. The algebraic dual of X, denoted by X^*, is the vector space of all linear functionals of X. If in addition X is a topological vector space, then the topological dual (or briefly 'dual') of X, denoted by X', is the vector space of all continuous linear functionals on X. Following Schaefer [28], Ch.IV, Sec.3, if X, Z are vector spaces over a field, a locally convex topology $\mathcal{T}$ on X is called 'consistent' with the duality $< X, Z >$, if the dual of $(X, \mathcal{T})$ is identical with Z (Z being viewed as a subspace of the algebraic dual X^*). We let $\sigma(Z, X)$ to denote the weak topology on X generated by Z.

The following theorem is the 'Mackey-Arens' theorem as in Schaefer [28], Ch.IV, Sec.3.

Theorem 5.1. *There is a finest locally convex topology $\tau(X, Z)$ on X consistent with $< X, Z >$. This topology is the topology of uniform convergence on all $\sigma(Z, X)$-compact convex circled subsets of Z.*

This topology on X is called the 'Mackey Topology' on X with respect to the dual pair $< X, Z >$. A locally convex space is called 'Mackey Space' if its topology is the Mackey topology.

Remark 5.1. *(a). Combining the results in Schaefer [28], Ch.IV,* 3.4 *and* 6.1 *we can easily conclude that, if $(X, \mathcal{T})$ is a metrizable space, then its Mackey topology is the topology of uniform convergence on all $\sigma(Z, X)$-compact subsets of Z. Furthermore, this topology coincides with $\mathcal{T}$.*

(b). Using the construction of projective limits and Schaefer [28], Ch.II, 5.3*, it is elementary to show that the projective limit of Fréchet spaces is a Fréchet space.*

As we have noted earlier, $(S_{F_i}, \mathcal{T}_i)$ could be endowed with the discrete topology, henceforth, we can suppose (w.l.o.g.) that for all $i \in \mathbb{N}$, $(S_{F_i}, \mathcal{T}_i)$ and $(S'_{F_i}, \mathcal{L}_i)$ are complete metric spaces (Fréchet spaces) themselves.

From Remark 5.1(a) we obtain the following:

Lemma 5.1. *For all $i \in \mathbb{N}$ the Mackey topologies on $(S_{F_i}, \mathcal{T}_i)$ and on $(S'_{F_i}, \mathcal{L}_i)$ coincide with the metric topologies, $\mathcal{T}_i$ and $\mathcal{L}_i$ respectively.*

The next theorem is of importance, since it describes exactly the topological framework of the duality of the processes 1, 2. It is a straightforward consequence of the result Schaefer [28], *Ch.IV*, 4.4, once we use Remark 5.1 and Lemma 5.1.

Theorem 5.2. *The topological dual of the reduced projective limit $S_{F_\infty} = \varprojlim \mu_{ij}(S_{F_i}, \mathcal{T}_i)$, under its metric topology, can be identified with the inductive limit of the family $\{S'_{F_i}, \mathcal{L}_i\}_{i \in I}$ with respect to the adjoint mappings ϕ_{ij} of μ_{ij}. That is $S'_{F_\infty} = \varinjlim \phi_{ij}(S'_{F_i}, \mathcal{L}_i)$.*

In this section we have provided the precise mathematical framework within which the convergence of the sequence in part (b) is comprehended. Moreover, we described precisely the topologies and the duality as of part (c) of the Main Claim.

6 Conclusion

In this article we have provided a mathematical model that describes the dual processes of:

1. The participation/partaking-methexis of the many particulars predicated as F to the Form F ('identified' with its essence F-ness), according to their degree of participation to it.

2. The presence-parousia of the Form-essence F to the particulars predicated as F, in analogy to their degree of participation to F as in 1.

In order to achieve this, we have provided and analyzed textual evidence from Plato's works that led to the construction of:

(i) An increasing infinite sequence $\{F_i\}_{i=0}^{\infty}$ of degrees of participation of the particulars predicated as F to the Form-essence F.

(ii) A decreasing sequence of sets of particulars $\{S_{F_i}\}_{i=0}^{\infty}$ according to the degrees of participation to the Form-essence F, as in (i). As we have shown, this sequence was conceived by Plato as convergent to the unique Form-essence F.

The topological framework regarding the existence and convergence of this sequence is based on the dual pair of Projective and Inductive topologies. We have argued that this model, even though it uses tools from the theory of Topological Vector Spaces, it is a solid and efficient model for comprehending the nature of the dual processes $1, 2$.

Competing Interests

The author declares that no competing interests exist.

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Note: The original texts, as well as their translations (unless stated otherwise) have been taken form the electronic database 'Thesaurus Linguae Graecae'.

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