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A Delayed Monod Chemostat Competition Model with Pulsed Input and Inhibitor in Polluted Environment

Manji Sang[1] and Jian-wen Jia[*1]

[1] *School of Mathematics and Computer Science, Shanxi Normal University , Shanxi, Linfen, 041004, P.R. China.*

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Abstract

In this paper, a two microorganisms and two nutrient chemostat competitive model with time delay and impulsive effect is considered. Besides, a polluted environment and an inhibitor were considered in this model. By using the theorem of the impulsive differential equations and delay differential equations, we obtain the sufficient conditions for the global attractivity of the microorganisms extinction periodic solution and the permanence of the system. Finally, the numerical simulations are presented for verifying the theoretical conclusions.

Keywords: Chemostat; Delay; Impulse; Global attractivity; Permanence.
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**Corresponding author: E-mail: jiajw.2008@163.com*

1 Introduction

Chemostat is a laboratory apparatus used to continuously cultivate microorganisms, and it is similar to a simplified model of the natural ecological environment. Chemostat culture is one of the most common of continuous cultivation. With the deeper research and more consideration of the chemostat culture, the chemostat plays a very important role in commercial microbial production, waste water treatment, bio-pharmaceutical, food processing and the other fields.

Due to the industrial pollution, agriculture pesticide and the other factors in nowadays, the survival environment of microbial population is polluted by toxic substances. We must estimate the degree of population damage by toxic substances for the purpose of using and dominating toxic substances reasonably. Therefore, it is very important to research the influence of the process of microbial culture polluted by toxic substances. It is very useful to find the theoretical threshold that can determine the persistent and extinction of the microorganisms. In consideration of the existence of this problem, articles [1-3] studied the mathematical model of microbial culture in polluted environment, they obtain the permanence for system and the existence and stability of the periodic solutions of microbial extinction.

The exploration and research of the dual nutrient chemostat model have been studied in articles [4-8]. The author distinguished two kinds of important double nutrient medium: complementary type and alternative type in article [4]. Complementary nutrient medium refers to the different kinds of necessary nutrient medium independently when microbial growth. Such as, carbon and nitrogen are two complementary nutrient medium for bacteria growth. Silica and phosphorus are two complementary nutrient medium for algae growth. The research of complementary dual nutrient medium chemostat model have been studied in articles [4-7]. Alternative nutrient medium refers to the same kinds of essential nutrient medium can substitute for each other when microbial growth. It depends on each other for the growth of microorganisms. Such as, two kinds of carbon resource can be substituted. Two kinds of phosphorus resource can be substituted. The research of alternative dual nutrient medium chemostat model have been studied in articles [8].

The authors in articles [9-11] discussed the chemostat model with inhibitor, and concluded some related results.

Microorganism intake of nutrient can not immediately translate into microorganism when the environment is changing. In other words, it is a time-lag process from intake nutrient to translate into microorganism. Therefore, it is more practical that delay and impulse input hazardous substance are considered in mathematical model research. When we want to control one kind of microbial growth, we can bring in inhibitor. In this paper, based on the articles [1] and [2], we build a model with complementary dual nutrient medium:

$$\begin{cases} \left.\begin{aligned} \frac{du(t)}{dt} &= -Du(t) - \frac{\gamma_1 x(t)u(t)}{\delta_1(k_1+u(t))} - \frac{\gamma_2 y(t)u(t)}{\delta_1(k_1+u(t))} \\ \frac{dv(t)}{dt} &= -Dv(t) - \frac{\gamma_3 x(t)v(t)}{\delta_2(k_2+v(t))} - \frac{\gamma_4 y(t)v(t)}{\delta_2(k_2+v(t))} \\ \frac{dx(t)}{dt} &= \exp(-D\tau_1 - \mu p(t-\tau_1))\frac{\gamma_1 x(t-\tau_1)u(t-\tau_1)}{k_1+u(t-\tau_1)} \\ &\quad + \exp(-D\tau_1 - \mu p(t-\tau_1))\frac{\gamma_3 x(t-\tau_1)v(t-\tau_1)}{k_2+v(t-\tau_1)} \\ &\quad -(D + r_1 c(t) + r_{12} y(t))x(t) \\ \frac{dy(t)}{dt} &= \exp(-D\tau_2)\frac{\gamma_2 y(t-\tau_2)u(t-\tau_2)}{k_1+u(t-\tau_2)} + \exp(-D\tau_2)\frac{\gamma_4 y(t-\tau_2)v(t-\tau_2)}{k_2+v(t-\tau_2)} \\ &\quad -(D + r_2 c(t) + r_{21} x(t))y(t) \\ \frac{dc(t)}{dt} &= -Dc(t) \\ \frac{dp(t)}{dt} &= -Dp(t) \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} &\Delta u(t) = \alpha u_0,\ \Delta v(t) = \alpha v_0,\ \ \Delta x(t) = 0, \\ &\Delta y(t) = 0 \ \ \Delta c(t) = \alpha c_0,\ \ \Delta p(t) = \alpha p_0. \end{aligned}\right\} t = nT, n \in N \end{cases} \tag{1.1}$$

where $u(t), v(t)$ represent the concentration of limiting substrate at time t, respectively; $x(t), y(t)$ represent the concentration of microorganisms in chemostat at time t, respectively; $c(t)$, $p(t)$ represent the concentration of the toxicant and the inhibitor at time t, respectively; $\alpha u_0, \alpha v_0$ denote the input capacity of two nutrient medium in each pulse of the T moment; $\alpha c_0, \alpha p_0$ denote the input capacity of the toxicant and the inhibitor in each pulse of the T moment; $D(0 \leq D < 1)$ is the dilution rate; $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are the predation constants of microorganisms; δ_1, δ_2 denote the substrate depletion rate, $T = \frac{\alpha}{D}$ is the period of the pulsing; r_1, r_2 are the depletion rate coefficients of the microorganisms population due to pollutant concentration, respectively; r_{12}, r_{21} are the competition coefficients of the microorganisms; τ_1, τ_2 denote the delayed time of nutrient solution to the microbial transformation, respectively; $\mathrm{e}^{-\mu p(t)}$ denotes the inhibitor on the degree of inhibition of $x(t)$; $\mu > 0$.

Considering the actual biological significance, we assume that the solutions of system (1.1) satisfying the initial conditions:

$$\begin{aligned} &(\phi_1(s), \phi_2(s), \phi_3(s), \phi_4(s), \phi_5(s), \phi_6(s)) \in C_+ = C([-\tau, 0], R_+^6), \phi_i(0) > 0, \\ &(i = 1, 2, 3, 4, 5, 6). \end{aligned} \tag{1.2}$$

2 Preliminary Results

Definition2.1 If $\lim\limits_{t\to\infty} x(t) = 0$, then $x(t)$ of system (1.1) is said to be extinction.

Definition2.2[12] The system (1.1) is uniformly persistent existence if there exist constants $M \geq m > 0$ such that for every positive solution $(u(t), v(t), x(t), y(t), c(t), p(t))$ of (1.1) with initial condition (1.2) satisfies $m \leq u(t), v(t), x(t), y(t), c(t), p(t) \leq M$ for all large t.

Lemma2.1[13] Considering the following delay differential equation

$$\frac{dz(t)}{dt} = r_1 z(t-\tau) - r_2 z(t),$$

where r_1, r_2, τ are positive numbers; $z(t) > 0$, $t \in [-\tau, 0]$. We have

(i) If $r_1 < r_2$, then $\lim\limits_{t\to\infty} z(t) = 0$; (ii) If $r_1 > r_2$, then $\lim\limits_{t\to\infty} z(t) = +\infty$.

For convenience, we give out the basic properties of the system:

$$\begin{cases} \frac{du(t)}{dt} = -Du(t), & t \neq nT, n \in N, \\ \triangle u = \alpha u_0, & t = nT, n \in N, \\ u(0^+) = u_{10} \geq 0. \end{cases} \tag{2.1}$$

Lemma 2.2[14] System (2.1) has a positive periodic solution $\widetilde{u}(t)$, and for every solution $u(t)$ of (2.1) which satisfies $u(0^+) = u_{10} \geq 0$, we have $|u(t) - \widetilde{u}(t)| \to 0$ as $t \to \infty$, besides

(i) If $u_{10} \geq \frac{\alpha u_0}{1-\exp(-DT)}$, then $u(t) \geq \widetilde{u}(t)$,

(ii) If $u_{10} < \frac{\alpha u_0}{1-\exp(-DT)}$, then $u(t) < \widetilde{u}(t)$,

where $\widetilde{u}(t) = \frac{\alpha u_0 \exp(-D(t-nT))}{1-\exp(-DT)}, t \in (nT, (n+1)T], n \in N, \widetilde{u}(0^+) = \frac{\alpha u_0}{1-\exp(-DT)}$.

Lemma 2.3 Let $X(t) = (u(t), v(t), x(t), y(t), c(t), p(t))$ be any solution of system (1.1) with initial condition (1.2), then there exist constant $M > 0$ and sufficient small $\varepsilon > 0$, such that $u(t),\ v(t) \leq M$, $x(t),\ y(t) \leq M$ and $0 < m_3 \leq c(t) \leq M_3, 0 < m_4 \leq p(t) \leq M_4$, where $m_3 = \frac{\alpha c_0 \exp(-DT)}{1-\exp(-DT)} - \varepsilon$, $M_3 = \frac{\alpha c_0}{1-\exp(-DT)} + \varepsilon,\ m_4 = \frac{\alpha p_0 \exp(-DT)}{1-\exp(-DT)} - \varepsilon,\ M_4 = \frac{\alpha p_0}{1-\exp(-DT)} + \varepsilon$ for all t large enough.

Proof Let $X(t) = (u(t), v(t), x(t), y(t), c(t), p(t))$ be any solution of system (1.1) with initial condition (1.2). Define a function $W(t) = \delta_1 u(t) + \delta_2 v(t) + \exp(D\tau_1) x(t+\tau_1) + \exp(D\tau_2) y(t+\tau_2)$. Then we calculate the right derivative of $W(t)$ along with the trajectory of system (1.1),

$$\begin{aligned} D^+W(t) = \quad & -DW(t) - (1 - \exp(-\mu p(t)))(\tfrac{r_1 x(t) u(t)}{k_1 + u(t)} + \tfrac{r_3 x(t) v(t)}{k_2 + v(t)}) \\ & - \exp(D\tau_1)[r_1 c(t+\tau_1) + r_{12} y(t+\tau_1)] x(t+\tau_1) \\ & - \exp(D\tau_2)[r_2 c(t+\tau_2) + r_{21} x(t+\tau_2)] y(t+\tau_2) \\ \leq \quad & -DW(t). \end{aligned}$$

By the impulsive differential inequality, we have

$$\begin{aligned} W(t) &= W(0^+)\exp(-Dt) + \tfrac{\alpha(\delta_1 u_0+\delta_2 v_0)\exp(-D(t-nT))}{1-\exp(-DT)} + \tfrac{\alpha(\delta_1 u_0+\delta_2 v_0)\exp(-Dt)}{\exp(-DT)-1} \\ &\le W(0^+)\exp(-Dt) + \tfrac{\alpha(\delta_1 u_0+\delta_2 v_0)\exp(-D(t-T))}{1-\exp(DT)} + \tfrac{\alpha(\delta_1 u_0+\delta_2 v_0)\exp(DT)}{\exp(DT)-1} \\ &\to \tfrac{\alpha(\delta_1 u_0+\delta_2 v_0)\exp(DT)}{\exp(DT)-1} = M, \;\; as\; t\to\infty, \end{aligned}$$

for all $t \in (nT,(n+1)T],\ n\in N$.

From the definition of $W(t)$, we know that there exist constant $M>0$ which make any solution $X(t)$ of (1.1) have $u(t),\ v(t),\ x(t),\ y(t)\le M$ for sufficient large t.

From Lemma 2.2 it follows that $\widetilde{c}(t) = \frac{\alpha c_0 \exp(-D(t-nT))}{1-\exp(-DT)},\ t\in(nT,(n+1)T]$, where $c(0) = \frac{\alpha c_0}{1-\exp(-DT)}$ is a positive periodic solution for system

$$\begin{cases} \frac{dc(t)}{dt} = -Dc(t), & t\neq nT, n\in N, \\ \triangle c = \alpha c_0, & t = nT, n\in N, \\ c(0^+) = c_{10}\ge 0, \end{cases} \tag{2.2}$$

which is global asymptotically stable. $\widetilde{p}(t) = \frac{\alpha p_0 \exp(-D(t-nT))}{1-\exp(-DT)},\ t\in(nT,(n+1)T]$, where $p(0) = \frac{\alpha p_0}{1-\exp(-DT)}$ is a positive periodic solution for system

$$\begin{cases} \frac{dp(t)}{dt} = -Dp(t), & t\neq nT, n\in N, \\ \triangle p = \alpha p_0, & t = nT, n\in N, \\ p(0^+) = p_{10}\ge 0, \end{cases} \tag{2.3}$$

which is global asymptotically stable. Then

$$\frac{\alpha c_0\exp(-DT)}{1-\exp(-DT)} \le \widetilde{c}(t) \le \frac{\alpha c_0}{1-\exp(-DT)}, t\ge 0;$$

$$\frac{\alpha p_0\exp(-DT)}{1-\exp(-DT)} \le \widetilde{p}(t) \le \frac{\alpha p_0}{1-\exp(-DT)}, t\ge 0.$$

From Lemma 2.2, suppose $c(t), p(t)$ are solution of system (2.2), (2.3)respectively, we have

$$0 < m_3 = \frac{\alpha c_0\exp(-DT)}{1-\exp(-DT)} - \varepsilon \le c(t) \le \frac{\alpha c_0}{1-\exp(-DT)} + \varepsilon = M_3,$$

$$0 < m_4 = \frac{\alpha p_0\exp(-DT)}{1-\exp(-DT)} - \varepsilon \le p(t) \le \frac{\alpha p_0}{1-\exp(-DT)} + \varepsilon = M_4,$$

for arbitrary $\varepsilon>0$ sufficiently small and t sufficiently large. We completed the proof.

2.1 Global attractivity

Microbial population's extinction in chemostat means microorganism disappeared from the chemostat completely, that is $\lim_{t\to\infty} x(t) = 0,\ \lim_{t\to\infty} y(t) = 0$. From Lemma 2.3 and the third and the fourth equations of system (1.1), we can yield

$$\frac{dx(t)}{dt} \le (\gamma_1+\gamma_3)\exp(-D\tau_1)x(t-\tau_1) - (D+r_1m_3)x(t),$$

$$\frac{dy(t)}{dt} \le (\gamma_2+\gamma_4)\exp(-D\tau_2)y(t-\tau_2) - (D+r_2m_3)y(t),$$

where $m_3 = \frac{\alpha c_0\exp(-DT)}{1-\exp(-DT)} - \varepsilon$. Obviously, if

$$(\gamma_1+\gamma_3)e^{-D\tau_1} < D + r_1\frac{\alpha c_0 e^{-DT}}{1-e^{-DT}},\quad (\gamma_2+\gamma_4)e^{-D\tau_2} < D + r_2\frac{\alpha c_0 e^{-DT}}{1-e^{-DT}}.$$

Then

$$(\gamma_1+\gamma_3)e^{-D\tau_1}<D+r_1m_3-\varepsilon,\quad (\gamma_2+\gamma_4)e^{-D\tau_2}<D+r_2m_3-\varepsilon,$$

for arbitrary $\varepsilon>0$ sufficiently small.

According to Lemma 2.1, we know $\lim_{t\to\infty}x(t)=0,\ \ \lim_{t\to\infty}y(t)=0,$ which means microbial population become extinct finally. No matter how much nutrient input, the number of microorganisms cultivated from the chemostat cannot compensate the outflow and killed by toxic substances. Thus, we assume

$$(\gamma_1+\gamma_3)e^{-D\tau_1}>D+r_1\frac{\alpha c_0e^{-DT}}{1-e^{-DT}},\quad (\gamma_2+\gamma_4)e^{-D\tau_2}>D+r_2\frac{\alpha c_0e^{-DT}}{1-e^{-DT}}.$$

From Lemma 2.2, system (1.1) has a microorganism-free periodic solution $(\widetilde{u}(t),\widetilde{v}(t),0,0,\widetilde{c}(t),\widetilde{p}(t))$, $t\in(nT,(n+1)T]$. Next, we will give the sufficient condition about global attractivity of the microorganism-free periodic solution.

Theorem 3.1 Let $(u(t),v(t),x(t),y(t),c(t),p(t))$ be any solution of system (1.1). If

$$\alpha u_0<\min\{\begin{array}{l}\frac{k_1}{\gamma_1}[D(1-e^{-DT}+r_1\alpha c_0e^{-DT}]\exp(D\tau_1+\frac{\mu\alpha p_0e^{-DT}}{1-e^{-DT}})-\frac{\gamma_3k_1}{\gamma_1k_2}\alpha v_0,\\ \frac{k_1}{\gamma_2}[D(1-e^{-DT})+r_2\alpha c_0e^{-DT}]e^{D\tau_2})-\frac{\gamma_4k_1}{\gamma_2k_2}\alpha v_0\}.\end{array}\tag{3.1}$$

or

$$\alpha v_0<\min\{\begin{array}{l}\frac{k_2}{\gamma_3}[D(1-e^{-DT})+r_1\alpha c_0e^{-DT}]\exp(D\tau_1+\frac{\mu\alpha p_0e^{-DT}}{1-e^{-DT}})-\frac{\gamma_1k_2}{\gamma_3k_1}\alpha u_0,\\ \frac{k_2}{\gamma_4}[D(1-e^{-DT})+r_2\alpha c_0e^{-DT}]e^{D\tau_2})-\frac{\gamma_2k_2}{\gamma_4k_1}\alpha u_0\}.\end{array}\tag{3.2}$$

Then the microorganism-free periodic solution $(\widetilde{u}(t),\widetilde{v}(t),0,0,\widetilde{c}(t),\widetilde{p}(t))$ of system (1.1) is global attractive.

Proof Let $(u(t),v(t),x(t),y(t),c(t),p(t))$ be any solution of system (1.1) with initial condition (1.2). From known condition (3.1) or (3.2), we conclude

$$\left\{\begin{array}{ll}\frac{\gamma_1}{k_1}\alpha u_0+\frac{\gamma_3}{k_2}\alpha v_0< & [D(1-e^{-DT})+r_1\alpha c_0e^{-DT}]\exp(D\tau_1+\frac{\mu\alpha p_0e^{-DT})}{1-e^{-DT}}),\\ \frac{\gamma_2}{k_1}\alpha u_0+\frac{\gamma_4}{k_2}\alpha v_0< & [D(1-e^{-DT})+r_2\alpha c_0e^{-DT}]e^{D\tau_2}).\end{array}\right.\tag{3.3}$$

We can choose a sufficient small positive constant ε such that

$$\left\{\begin{array}{ll}\frac{\gamma_1}{k_1}(\frac{\alpha u_0}{1-e^{-DT}}+\varepsilon)+\frac{\gamma_3}{k_2}(\frac{\alpha v_0}{1-e^{-DT}}+\varepsilon)< & [D+r_1(\frac{\alpha c_0\exp(-DT)}{1-\exp(-DT)}-\varepsilon)]\\ & \exp[D\tau_1+\mu(\frac{\alpha p_0\exp(-DT)}{1-\exp(-DT)}-\varepsilon)],\\ \frac{\gamma_2}{k_1}(\frac{\alpha u_0}{1-e^{-DT}}+\varepsilon)+\frac{\gamma_4}{k_2}(\frac{\alpha v_0}{1-e^{-DT}}+\varepsilon)< & [D+r_2(\frac{\alpha c_0\exp(-DT)}{1-\exp(-DT)}-\varepsilon)]\exp(D\tau_2).\end{array}\right.\tag{3.4}$$

Therefore,

$$\left\{\begin{array}{ll}\exp[D\tau_1+\mu(\frac{\alpha p_0\exp(-DT)}{1-\exp(-DT)}-\varepsilon)] & [\frac{\gamma_1}{k_1}(\frac{\alpha u_0}{1-e^{-DT}}+\varepsilon)+\frac{\gamma_3}{k_2}(\frac{\alpha v_0}{1-e^{-DT}}+\varepsilon)]\\ & <D+r_1(\frac{\alpha c_0\exp(-DT)}{1-\exp(-DT)}-\varepsilon),\\ \exp(D\tau_2)[\frac{\gamma_2}{k_1}(\frac{\alpha u_0}{1-e^{-DT}}+\varepsilon) & +\frac{\gamma_4}{k_2}(\frac{\alpha v_0}{1-e^{-DT}}+\varepsilon)]<D+r_2(\frac{\alpha c_0\exp(-DT)}{1-\exp(-DT)}-\varepsilon).\end{array}\right.\tag{3.5}$$

From system (1.1), we have $u'(t)\le -Du(t),\ v'(t)\le -Dv(t)$. Then we consider the comparison system

$$\left\{\begin{array}{ll}\left.\begin{array}{l}\frac{dS}{dt}=-DS\\ \frac{dR}{dt}=-DR\end{array}\right\} & t\neq nT,n\in N,\\ \Delta S=\alpha u_0,\ \ \Delta R=\alpha v_0 & t=nT,n\in N\end{array}\right.\tag{3.6}$$

From lemma 2.2, we obtain the periodic solution of system (3.6) ie. $\widetilde{S}(t)=\widetilde{u}(t)=\frac{\alpha u_0\exp(-D(t-nT))}{1-\exp(-DT)}$, $\widetilde{R}(t)=\widetilde{v}(t)=\frac{\alpha v_0\exp(-D(t-nT))}{1-\exp(-DT)},t\in(nT,(n+1)T],n\in N.$ which is global attractive. In other words, Suppose $(S(t), R(t))$ is a solution of system (3.6) with initial condition $S(0^+) = \alpha u_0, R(0^+) = \alpha v_0$. there exists $n_1 \in N$, such that

$$u(t) \le S(t) < \widetilde{S}(t) + \varepsilon, \;\; v(t) \le R(t) < \widetilde{R}(t) + \varepsilon, \;\; t > n_1 T,$$

for arbitrary $\varepsilon > 0$.

According to Lemma 2.3, we obtain that

$$u(t) < \widetilde{S}(t) + \varepsilon < \frac{\alpha u_0}{1 - e^{-DT}} + \varepsilon \overset{\triangle}{=} \eta, \;\; v(t) < \widetilde{R}(t) < \frac{\alpha v_0}{1 - e^{-DT}} + \varepsilon \overset{\triangle}{=} \eta_1. \tag{3.7}$$

From the third and the fourth equations of system (1.1), it follows that

$$\begin{cases} \frac{dx(t)}{dt} = & \exp(-D\tau_1 - \mu p(t-\tau_1))[\frac{\gamma_1 u(t-\tau_1)}{k_1 + u(t-\tau_1)} + \frac{\gamma_3 v(t-\tau_1)}{k_2 + v(t-\tau_1)}]x(t-\tau_1) \\ & -(D + r_1 c(t) + r_{12} y(t))x(t), \\ \frac{dy(t)}{dt} = & \exp(-D\tau_2)[\frac{\gamma_2 u(t-\tau_2)}{k_1 + u(t-\tau_2)} + \frac{\gamma_4 v(t-\tau_2)}{k_2 + v(t-\tau_2)}]y(t-\tau_2) - (D + r_2 c(t) + r_{21} x(t))y(t). \end{cases}$$

And then combine with (3.5), it follows that

$$\frac{dx(t)}{dt} \le \exp(-D\tau_1 - \mu m_4)(\frac{\gamma_1 \eta}{k_1} + \frac{\gamma_3 \eta_1}{k_2})x(t-\tau_1) - (D + r_1 m_3)x(t),$$

for $t > n_1 T + \tau_1$, and

$$\frac{dy(t)}{dt} \le \exp(-D\tau_2)(\frac{\gamma_2 \eta}{k_1} + \frac{\gamma_4 \eta_1}{k_2})y(t-\tau_2) - (D + r_2 m_3)y(t),$$

for $t > n_1 T + \tau_2$, where $\eta, \; \eta_1, \; m_3, \; m_4$ are defined before.

Consider the following comparison equations

$$\begin{cases} \frac{dz_1(t)}{dt} = \exp(-D\tau_1 - \mu m_4)(\frac{\gamma_1 \eta}{k_1} + \frac{\gamma_3 \eta_1}{k_2})z_1(t-\tau_1) - (D + r_1 m_3)z_1(t) \\ \frac{dz_2(t)}{dt} = \exp(-D\tau_2)(\frac{\gamma_2 \eta}{k_1} + \frac{\gamma_4 \eta_1}{k_2})z_2(t-\tau_2) - (D + r_2 m_3)z_2(t) \end{cases}$$

From Lemma 2.1 and (3.5), it is easy to known that $\lim\limits_{t\to\infty} z_1 = 0, \;\; \lim\limits_{t\to\infty} z_2 = 0.$

If $s \in [-\tau_1, 0]$, then $x(s) = z_1(s) > 0$; if $r \in [-\tau_2, 0]$, then $y(r) = z_2(r) > 0$. Thus on the basis of comparison theorem of differential equation and solution's nonnegativity, we have $x(t) \to 0, \, y(t) \to 0$ as $t \to \infty$.

Because variable u, v, x and y are not appear in the third or the fourth equation of system (1.1), from Lemma 2.2, we have $c(t) \to \widetilde{c}(t), \, p(t) \to \widetilde{p}(t)$ as $t \to \infty$, where $\widetilde{c}(t) = \frac{\alpha c_0 \exp(-D(t-nT))}{1-\exp(-DT)}, \widetilde{p}(t) = \frac{\alpha p_0 \exp(-D(t-nT))}{1-\exp(-DT)}, \; t \in (nT, (n+1)T]$. We have completed the proof.

Corollary 3.1 Assume that

$$\alpha c_0 > \max\{ \begin{array}{l} \frac{1}{r_1}(\frac{\gamma_1}{k_1}\alpha u_0 + \frac{\gamma_3}{k_2}\alpha v_0)\exp(DT - D\tau_1 - \frac{\mu \alpha p_0 \exp(-DT)}{1-\exp(-DT)} - \frac{D}{r_1}(\exp(DT) - 1)), \\ \frac{1}{r_2}(\frac{\gamma_2}{k_1}\alpha u_0 + \frac{\gamma_4}{k_2}\alpha v_0)\exp(DT - D\tau_2) - \frac{D}{r_2}(\exp(DT) - 1)\}, \end{array}$$

then the microorganism-free periodic solution $(\widetilde{u}(t), \widetilde{v}(t), 0, 0, \widetilde{c}(t), \widetilde{p}(t))$ of system (1.1) is global attractive.

Corollary 3.2 Assume that

$$\begin{array}{l} \tau_1 > \frac{1}{D}\ln \frac{\frac{\gamma_1}{k_1}\alpha u_0 + \frac{\gamma_3}{k_2}\alpha v_0}{D(1-\exp(-DT)) + r_1 \alpha c_0 \exp(-DT)} - \frac{\mu \alpha p_0 \exp(-DT)}{D(1-\exp(-DT))}, \\ \tau_2 > \frac{1}{D}\ln \frac{\frac{\gamma_2}{k_1}\alpha u_0 + \frac{\gamma_4}{k_2}\alpha v_0}{D(1-\exp(-DT)) + r_2 \alpha c_0 \exp(-DT)}, \end{array}$$

then the microorganism-free periodic solution $(\widetilde{u}(t), \widetilde{v}(t), 0, 0, \widetilde{c}(t), \widetilde{p}(t))$ of system (1.1) is global attractive.

3 Persistent

Note that

$$\Re=\min\{\frac{(\alpha u_0+\alpha v_0)\gamma_{11}\exp(-D\tau_1-\mu M_4)}{(k+M)(D+r_1M_3+r_{12}M)[\exp(DT+\frac{\gamma_{24}MT}{\delta_{21}k_{21}})-1]},\ \frac{(\alpha u_0+\alpha v_0)\gamma_{22}\exp(-D\tau_2)}{(k+M)(D+r_2M_3+r_{21}M)[\exp(DT+\frac{\gamma_{13}MT}{\delta_{12}k_{12}})-1]}\},$$

where $\gamma_{11}=\min\{\gamma_1,\gamma_3\}$, $\gamma_{22}=\min\{\gamma_2,\gamma_4\}$, $k=\max\{k_1,k_2\}$, $\frac{\gamma_{13}}{\delta_{12}k_{12}}=\max\{\frac{\gamma_1}{\delta_1k_1},\frac{\gamma_3}{\delta_2k_2}\}$, $\frac{\gamma_{24}}{\delta_{21}k_{21}}=\max\{\frac{\gamma_2}{\delta_1k_1},\frac{\gamma_4}{\delta_2k_2}\}$.

Theorem 4.1 If $\Re>1$, then system (1.1) is persistent.

Proof Let $(u(t),v(t),x(t),y(t),c(t),p(t))$ be any solution of system (1.1) with initial condition (1.2).

Firstly, from the proof procedure of Lemma 2.3, it follows that there exist constant $M>0$ and t large enough, such that $u(t),\ v(t),\ x(t),\ y(t)\le M$ and $m_3\le c(t)\le M_3, m_4\le p(t)\le M_4$ for any solution $(u(t),v(t),x(t),y(t),c(t),p(t))$ of system (1.1).

Secondly, we will prove $u(t),v(t)$ have positive lower bound.

From the first and the second equations of system (1.1), we have

$$\begin{cases}\frac{du(t)}{dt}\ge -Du(t)-\frac{\gamma_1Mu(t)}{\delta_1k_1}-\frac{\gamma_2Mu(t)}{\delta_1k_1},\\ \frac{dv(t)}{dt}\ge -Dv(t)-\frac{\gamma_3Mv(t)}{\delta_2k_2}-\frac{\gamma_4Mv(t)}{\delta_2k_2}.\end{cases}$$

Consider the following impulsive comparison system

$$\begin{cases}\left.\begin{array}{l}\frac{dS}{dt}=-(D+\frac{\gamma_1M}{\delta_1k_1}+\frac{\gamma_2M}{\delta_1k_1})S(t)\\ \frac{dR}{dt}=-(D+\frac{\gamma_3M}{\delta_2k_2}+\frac{\gamma_4M}{\delta_2k_2})R(t)\end{array}\right\}, & t\ne nT,\\ \Delta S(t)=\alpha u_0,\ \ \Delta R(t)=\alpha v_0, & t=nT.\end{cases}\tag{4.1}$$

The positive periodic solution of (4.1) is:

$$\begin{array}{ll}\widetilde{S}(t)=\frac{\alpha u_0\exp(-(D+\frac{\gamma_1M}{\delta_1k_1}+\frac{\gamma_2M}{\delta_1k_1})(t-nT))}{1-\exp(-(D+\frac{\gamma_1M}{\delta_1k_1}+\frac{\gamma_2M}{\delta_1k_1})T)}, & \\ & t\in(nT,(n+1)T].\\ \widetilde{R}(t)=\frac{\alpha v_0\exp(-(D+\frac{\gamma_3M}{\delta_2k_2}+\frac{\gamma_4M}{\delta_2k_2})(t-nT))}{1-\exp(-(D+\frac{\gamma_3M}{\delta_2k_2}+\frac{\gamma_4M}{\delta_2k_2})T)}, & \end{array}$$

Using the comparison theorem, there exist sufficient small positive numbers $\varepsilon_1,\varepsilon_2$ such that

$$u(t)\ge S(t)>\widetilde{S}(t)-\varepsilon_1>\frac{\alpha u_0\exp(-(D+\frac{\gamma_1m_x}{\delta_1k_1}+\frac{\gamma_2M}{\delta_1k_1})T)}{1-\exp(-(D+\frac{\gamma_1M}{\delta_1k_1}+\frac{\gamma_2M}{\delta_1k_1})T)}-\varepsilon_1\triangleq m_1,$$

$$v(t)\ge R(t)>\widetilde{R}(t))-\varepsilon_2>\frac{\alpha v_0\exp(-(D+\frac{\gamma_3m_x}{\delta_2k_2}+\frac{\gamma_4M}{\delta_2k_2})T)}{1-\exp(-(D+\frac{\gamma_3M}{\delta_2k_2}+\frac{\gamma_4M}{\delta_2k_2})T)}-\varepsilon_2\triangleq m_2.$$

From mentioned above, if t is large enough, then $u(t)\ge m_1, v(t)\ge m_2$.

Next, we will prove $x(t),y(t)$ have positive lower bound.

Note that the third and the fourth equations of system (1.1) can be rewritten as

$$\begin{cases}\frac{dx(t)}{dt}= & [(\frac{\gamma_1u(t)}{k_1+u(t)}+\frac{\gamma_3v(t)}{k_2+v(t)})\exp(-D\tau_1-\mu p(t)-(D+r_1c(t)+r_{12}y(t))]x(t)\\ & -\exp(-D\tau_1)\frac{d}{dt}\int_{t-\tau_1}^{t}(\frac{\gamma_1u(\theta)}{k_1+u(\theta)}+\frac{\gamma_3v(\theta)}{k_2+v(\theta)})x(\theta)\exp(-\mu p(\theta))d\theta,\\ \frac{dy(t)}{dt}= & [(\frac{\gamma_2u(t)}{k_1+u(t)}+\frac{\gamma_4v(t)}{k_2+v(t)})\exp(-D\tau_2)-(D+r_2c(t)+r_{21}x(t))]y(t)\\ & -\exp(-D\tau_2)\frac{d}{dt}\int_{t-\tau_2}^{t}(\frac{\gamma_2u(\theta)}{k_1+u(\theta)}+\frac{\gamma_4v(\theta)}{k_2+v(\theta)})y(\theta)d\theta,\end{cases}\tag{4.2}$$

Define

$$V_1(t)=x(t)+\exp(-D\tau_1)\int_{t-\tau_1}^{t}(\frac{\gamma_1u(\theta)}{k_1+u(\theta)}+\frac{\gamma_3v(\theta)}{k_2+v(\theta)})x(\theta)\exp(-\mu p(\theta))d\theta,$$

$$V_2(t) = y(t) + \exp(-D\tau_2)\int_{t-\tau_2}^{t}(\frac{\gamma_2 u(\theta)}{k_1+u(\theta)} + \frac{\gamma_4 v(\theta)}{k_2+v(\theta)})y(\theta)d\theta,$$

We calculate the derivative of $V_1(t), V_2(t)$ along the solutions of system (1.1).

$$\begin{array}{ll} \frac{dV_1(t)}{dt} & = [(\frac{\gamma_1 u(t)}{k_1+u(t)} + \frac{\gamma_3 v(t)}{k_2+v(t)})\exp(-D\tau_1 - \mu p(t)) - (D + r_1 c(t) + r_{12}y(t))]x(t) \\ & \geq (D + r_1 M_3 + r_{12}M)[\frac{\exp(-D\tau_1 - \mu M_4)}{D+r_1M_3+r_{12}M}(\frac{\gamma_1 u(t)}{k_1+u(t)} + \frac{\gamma_3 v(t)}{k_2+v(t)}) - 1]x(t), \\ \frac{dV_2(t)}{dt} & = [(\frac{\gamma_2 u(t)}{k_1+u(t)} + \frac{\gamma_4 v(t)}{k_2+v(t)})\exp(-D\tau_2) - (D + r_2 c(t) + r_{21}x(t))]y(t) \\ & \geq (D + r_2 M_3 + r_{21}M)[\frac{\exp(-D\tau_2)}{D+r_2M_3+r_{21}M}(\frac{\gamma_2 u(t)}{k_1+u(t)} + \frac{\gamma_4 v(t)}{k_2+v(t)}) - 1]y(t). \end{array} \quad (4.3)$$

Since $\Re > 1$, we can conclude

$$\frac{1}{T}\ln(\frac{(\alpha u_0 + \alpha v_0)\gamma_{11}\exp(-D\tau_1 - \mu M_4)}{(k+M)(D + r_1M_3 + r_{12}M)} + 1) - D - \frac{\gamma_{24}}{\delta_{21}k_{21}}M > 0,$$

$$\frac{1}{T}\ln(\frac{(\alpha u_0 + \alpha v_0)\gamma_{22}\exp(-D\tau_2)}{(k+M)(D + r_2M_3 + r_{21}M)} + 1) - D - \frac{\gamma_{13}}{\delta_{12}k_{12}}M > 0.$$

So we can get

$$\frac{\delta_{12}k_{12}}{\gamma_{13}}[\frac{1}{T}\ln(\frac{(\alpha u_0 + \alpha v_0)\gamma_{11}\exp(-D\tau_1 - \mu M_4)}{(k+M)(D + r_1M_3 + r_{12}M)} + 1) - D - \frac{\gamma_{24}M}{\delta_{21}k_{21}}M] > 0,$$

$$\frac{\delta_{21}k_{21}}{\gamma_{24}}[\frac{1}{T}\ln(\frac{(\alpha u_0 + \alpha v_0)\gamma_{22}\exp(-D\tau_2)}{(k+M)(D + r_2M_3 + r_{21}M)} + 1) - D - \frac{\gamma_{13}}{\delta_{12}k_{12}}M] > 0.$$

Therefore, we have $m_x > 0, m_y > 0$ such that

$$0 < m_x < \frac{\delta_{12}k_{12}}{\gamma_{13}}[\frac{1}{T}\ln(\frac{(\alpha u_0 + \alpha v_0)\gamma_{11}\exp(-D\tau_1 - \mu M_4)}{(k+M)(D + r_1M_3 + r_{12}M)} + 1) - D - \frac{\gamma_{24}M}{\delta_{21}k_{21}}M],$$

$$0 < m_y < \frac{\delta_{21}k_{21}}{\gamma_{24}}[\frac{1}{T}\ln(\frac{(\alpha u_0 + \alpha v_0)\gamma_{22}\exp(-D\tau_2)}{(k+M)(D + r_2M_3 + r_{21}M)} + 1) - D - \frac{\gamma_{13}}{\delta_{12}k_{12}}M].$$

We assert that, there exist $m_x > 0, m_y > 0$ such that $x(t) \geq m_x, y(t) \geq m_y$ for all large t. Then we will prove it in two steps.

Step1 We prove that there exist $t_1 > 0, t_2 > 0$, such that $x(t_1) \geq m_x, y(t_2) \geq m_y$. Otherwise, there will be three cases:

(i)There exists $t_2 > 0$, such that $y(t_2) \geq m_y$, and for all $t > 0$, $x(t) < m_x$ is valid;

(ii)There exists $t_1 > 0$, such that $x(t_1) \geq m_x$, and for all $t > 0$, $y(t) < m_y$ is valid;

(iii)For all $t > 0$, $x(t) < m_x, y(t) < m_y$ is valid.

For case (i),

$$\begin{cases} \frac{du(t)}{dt} \geq -(D + \frac{\gamma_1 m_x}{\delta_1 k_1} + \frac{\gamma_2 M}{\delta_1 k_1})u(t), \\ \frac{dv(t)}{dt} \geq -(D + \frac{\gamma_3 m_x}{\delta_2 k_2} + \frac{\gamma_4 M}{\delta_2 k_2})v(t), \end{cases}$$

According to the comparison theorem of impulsive differential equations, there exists a $T_1 > 0$ and we can choose sufficient small positive numbers $\varepsilon_3, \varepsilon_4$ such that

$$\begin{cases} u(t) \geq \widetilde{S}_1(t) - \varepsilon_3 > \frac{\alpha u_0 \exp(-(D + \frac{\gamma_1 m_x}{\delta_1 k_1} + \frac{\gamma_2 M}{\delta_1 k_1})T)}{1 - \exp(-(D + \frac{\gamma_1 m_x}{\delta_1 k_1} + \frac{\gamma_2 M}{\delta_1 k_1})T)} - \varepsilon_3 \triangleq \eta_2, \\ v(t) \geq \widetilde{R}_1(t) - \varepsilon_4 > \frac{\alpha v_0 \exp(-(D + \frac{\gamma_3 m_x}{\delta_2 k_2} + \frac{\gamma_4 M}{\delta_2 k_2})T)}{1 - \exp(-(D + \frac{\gamma_3 m_x}{\delta_2 k_2} + \frac{\gamma_4 M}{\delta_2 k_2})T)} - \varepsilon_4 \triangleq \eta_3, \end{cases} \quad (4.4)$$

for all $t > T_1 + \tau_1$, where

$$\begin{cases} \widetilde{S}_1(t) = \frac{\alpha u_0 \exp(-(D + \frac{\gamma_1 m_x}{\delta_1 k_1} + \frac{\gamma_2 M}{\delta_1 k_1})(t - nT))}{1 - \exp(-(D + \frac{\gamma_1 m_x}{\delta_1 k_1} + \frac{\gamma_2 M}{\delta_1 k_1})T)}, \\ \widetilde{R}_1(t) = \frac{\alpha v_0 \exp(-(D + \frac{\gamma_3 m_x}{\delta_2 k_2} + \frac{\gamma_4 M}{\delta_2 k_2})(t - nT))}{1 - \exp(-(D + \frac{\gamma_3 m_x}{\delta_2 k_2} + \frac{\gamma_4 M}{\delta_2 k_2})T)}, \end{cases} \quad t \in (nT, (n+1)T].$$

Then we have

$$\frac{dV_1(t)}{dt} \geq (D + r_1M_3 + r_{12}M)[\frac{\exp(-D\tau_1 - \mu M_4)}{D + r_1M_3 + r_{12}M}(\frac{\gamma_1\eta_2}{k_1 + M} + \frac{\gamma_3\eta_3}{k_2 + M}) - 1]x(t), \quad t > T_1. \quad (4.5)$$

Because of

$$m_x < \frac{\delta_{12}k_{12}}{\gamma_{13}}[\frac{1}{T}\ln(\frac{(\alpha u_0 + \alpha v_0)\gamma_{11}\exp(-D\tau_1 - \mu M_4)}{(k + M)(D + r_1M_3 + r_{12}M)} + 1) - D - \frac{\gamma_{24}M}{\delta_{21}k_{21}}M],$$

we can get $\frac{\gamma_{11}}{k+M}\frac{\alpha u_0+\alpha v_0}{\exp((D+\frac{\gamma_{13}m_x}{\delta_{12}k_{12}}+\frac{\gamma_{24}M}{\delta_{21}k_{21}})T-1)} > \frac{D+r_1M_3+r_{12}M}{\exp(-D\tau_1-\mu M_4)}$, next $\frac{\gamma_1}{k_1+M}\frac{\alpha u_0}{\exp((D+\frac{\gamma_1 m_x}{\delta_1 k_1}+\frac{\gamma_2 M}{\delta_1 k_1})T)-1} + \frac{\gamma_3}{k_2+M}\frac{\alpha v_0}{\exp((D+\frac{\gamma_3 m_x}{\delta_2 k_2}+\frac{\gamma_4 M}{\delta_2 k_2})T)-1} > \frac{D+r_1M_3+r_{12}M}{\exp(-D\tau_1-\mu M_4)}$.

Thus, there exist sufficient small positive numbers $\varepsilon_3, \varepsilon_4$ such that

$$\frac{\exp(-D\tau_1 - \mu M_4)}{D + r_1M_3 + r_{12}M}(\frac{\gamma_1\eta_2}{k_1 + M} + \frac{\gamma_3\eta_3}{k_2 + M}) > 1. \quad (4.6)$$

Let $x_1^* = \min_{t\in[T_1,T_1+\tau_1]} x(t)$. We assert that for all $t \geq T_1$, $x(t) \geq x_1^*$ is valid. Otherwise, there exists a nonnegative constant T_2, such that $x(t) \geq x_1^*, x(T_1 + \tau_1 + T_2) = x_1^*$, $x'(T_1 + \tau_1 + T_2) \leq 0$ for all $t \in [T_1, T_1 + \tau_1 + T_2]$. By the third equation of system (1.1) and (4.6), we can get

$$\begin{aligned} x'(T_1 + \tau_1 + T_2) &\geq \exp(-D\tau_1 - \mu M_4)(\tfrac{\gamma_1\eta_2 x_1^*}{k_1+M} + \tfrac{\gamma_3\eta_3 x_1^*}{k_2+M}) \\ &= (D + r_1M_3 + r_{12}M)[\tfrac{\exp(-D\tau_1-\mu M_4)}{D+r_1M_3+r_{12}M}(\tfrac{\gamma_1\eta_2}{k_1+M} + \tfrac{\gamma_3\eta_3}{k_2+M}) - 1]x_1^* > 0, \end{aligned} \quad (4.7)$$

that is the contradiction. Thus, we have $x(t) \geq x_1^* > 0$ for any $t \geq T_1$. From (4.5) and (4.7), we have

$$\frac{dV_1(t)}{dt} \geq (D + r_1M_3 + r_{12}M)[\frac{\exp(-D\tau_1 - \mu M_4)}{D + r_1M_3 + r_{12}M}(\frac{\gamma_1\eta_2}{k_1 + M} + \frac{\gamma_3\eta_3}{k_2 + M}) - 1]x_1^* > 0, \quad t > T_1.$$

This means, $V_1(t) \to +\infty$ as $t \to \infty$, that is a contradiction with the boundedness of $V_1(t) \leq M[1 + (\frac{\gamma_1}{k_1} + \frac{\gamma_3}{k_2})M\tau_1\exp(-D\tau_1 - \mu m_4)]$. Therefore, $x(t) < m_x$ can not be true for all $t > 0$.

In the same way, we can prove the case (ii).

Next for case (iii).

$$\begin{cases} \frac{du(t)}{dt} \geq -(D + \frac{\gamma_1 m_x}{\delta_1 k_1} + \frac{\gamma_2 m_y}{\delta_1 k_1})u(t), \\ \frac{dv(t)}{dt} \geq -(D + \frac{\gamma_3 m_x}{\delta_2 k_2} + \frac{\gamma_4 m_y}{\delta_2 k_2})v(t). \end{cases}$$

According to the comparison theorem of impulsive differential equations, there exists a $T_3 > 0$ and we can choose sufficient small positive numbers $\varepsilon_5, \varepsilon_6$ such that

$$\begin{cases} u(t) \geq \widetilde{S}_2(t) - \varepsilon_5 > \frac{\alpha u_0 \exp(-(D+\frac{\gamma_1 m_x}{\delta_1 k_1}+\frac{\gamma_2 m_y}{\delta_1 k_1})T)}{1-\exp(-(D+\frac{\gamma_1 m_x}{\delta_1 k_1}+\frac{\gamma_2 m_y}{\delta_1 k_1})T)} - \varepsilon_5 \triangleq \eta_4, \\ v(t) \geq \widetilde{R}_2(t) - \varepsilon_6 > \frac{\alpha v_0 \exp(-(D+\frac{\gamma_3 m_x}{\delta_2 k_2}+\frac{\gamma_4 m_y}{\delta_2 k_2})T)}{1-\exp(-(D+\frac{\gamma_3 m_x}{\delta_2 k_2}+\frac{\gamma_4 m_y}{\delta_2 k_2})T)} - \varepsilon_6 \triangleq \eta_5, \end{cases} \quad (4.8)$$

for all $t > T_3 + \tau_1$, where

$$\begin{cases} \widetilde{S}_2(t) = \frac{\alpha u_0 \exp(-(D+\frac{\gamma_1 m_x}{\delta_1 k_1}+\frac{\gamma_2 m_y}{\delta_1 k_1})(t-nT))}{1-\exp(-(D+\frac{\gamma_1 m_x}{\delta_1 k_1}+\frac{\gamma_2 M}{\delta_1 k_1})T)}, \\ \widetilde{R}_2(t) = \frac{\alpha v_0 \exp(-(D+\frac{\gamma_3 m_x}{\delta_2 k_2}+\frac{\gamma_4 m_y}{\delta_2 k_2})(t-nT))}{1-\exp(-(D+\frac{\gamma_3 m_x}{\delta_2 k_2}+\frac{\gamma_4 M}{\delta_2 k_2})T)}, \end{cases} \quad t \in (nT, (n+1)T].$$

Then we have

$$\frac{dV_1(t)}{dt} \geq (D + r_1M_3 + r_{12}M)[\frac{\exp(-D\tau_1 - \mu M_4)}{D + r_1M_3 + r_{12}M}(\frac{\gamma_1\eta_4}{k_1 + M} + \frac{\gamma_3\eta_5}{k_2 + M}) - 1]x(t), \ t > T_1. \quad (4.9)$$

From the known conditions, we have
$\frac{\gamma_1}{k_1+M}\frac{\alpha u_0}{\exp((D+\frac{\gamma_1 m_x}{\delta_1 k_1}+\frac{\gamma_2 m_y}{\delta_1 k_1})T)-1}+\frac{\gamma_3}{k_2+M}\frac{\alpha v_0}{\exp((D+\frac{\gamma_3 m_x}{\delta_2 k_2}+\frac{\gamma_4 m_y}{\delta_2 k_2})T)-1} > \frac{D+r_1M_3+r_{12}M}{\exp(-D\tau_1-\mu M_4)}$.

Therefore, there exists sufficient small positive numbers $\varepsilon_5, \varepsilon_6$ such that

$$\frac{\exp(-D\tau_1-\mu M_4)}{D+r_1M_3+r_{12}M}(\frac{\gamma_1\eta_4}{k_1+M}+\frac{\gamma_3\eta_5}{k_2+M})>1. \tag{4.10}$$

Thus

$$\frac{dV_1(t)}{dt}\geq(D+r_1M_3+r_{12}M)[\frac{\exp(-D\tau_1-\mu M_4)}{D+r_1M_3+r_{12}M}(\frac{\gamma_1\eta_4}{k_1+M}+\frac{\gamma_3\eta_5}{k_2+M})-1]x(t)>0,\ t>T_1.$$

Similar to the previous discussion, $V_1(t)\to+\infty$ as $t\to\infty$, that is contradiction.

Step2 On the one hand, if for all large t, $x(t)\geq m_1$ is valid, then the purpose achieved. On the other hand, $x(t)$ is oscillating on m_x. Let $m^*=\min\{\frac{m_x}{2}, m_x\exp(-(D+r_1M_3+r_{12}M)\tau_1)\}$. We assert $x(t)\geq m^*$ for all t large enough.

First, there exist two positive numbers $\widetilde{t},\omega$ such that $x(\widetilde{t})=x(\widetilde{t}+\omega)=m_x$ and $x(t)<m_x, \widetilde{t}<t<\widetilde{t}+\omega$.

Because $x(t)$ is continuous, uniformly bounded and not affected by impulse, we obtain that $x(t)$ is uniformly continuous. Therefore, there exists constant β(where $0<\beta<\tau_1$, β is not dependent on the choice of $\widetilde{t}$) for all $\widetilde{t}<t<\widetilde{t}+\beta$, such that $x(t)>\frac{m_x}{2}$.

If $\omega\leq\beta$, then conclusion is valid.

For $\beta<\omega\leq\tau_1$, by the third equation of system (1.1), we have $x'(t)\geq-(D+r_1M_3+r_{12}M)x(t)$ for $\widetilde{t}<t<\widetilde{t}+\omega$. Because $x(\widetilde{t})=m_x$, $x(t)\geq m_x\exp(-(D+r_1M_3+r_{12}M)\tau_1)$ is valid for $\widetilde{t}<t<\widetilde{t}+\omega\leq\widetilde{t}+\tau_1$. Obviously, if $\widetilde{t}<t<\widetilde{t}+\omega$ then $x(t)\geq m^*$.

For $\omega\geq\tau_1$, if $\widetilde{t}<t<\widetilde{t}+\tau_1$ then $x(t)\geq m^*$. The same discussion to above, we can get the conclusion $x(t)\geq m^*$ for $\widetilde{t}<t<\widetilde{t}+\omega$.

Because interval $[\widetilde{t},\widetilde{t}+\omega]$ is arbitrary(only $\widetilde{t}$ is large enough), we obtain $x(t)\geq m^*$ for all large t. From the above discussion we obtain that the choose of m^* is not dependent on the positive solution of system (1.1). So $x(t)$ has the lower bound m^*.

Evidenced by the same way, $y(t)\geq m^{**}$, where $m^{**}=\min\{\frac{m_y}{2}, m_y\exp(-(D+r_2M_3+r_{21}M)\tau_2)\}$. The proof of Theorem 4.1 is complete.

Corollary 4.1 System (1.1) is permanent, if one of the following conditions is satisfied

(i) $\alpha u_0+\alpha v_0>\max\{\frac{(k+M)(D+r_1M_3+r_{12}M)[\exp(DT+\frac{\gamma_{24}MT}{\delta_{21}k_{21}})-1]}{\gamma_{11}\exp(-D\tau_1-\mu M_4)}, \frac{(k+M)(D+r_2M_3+r_{21}M)[\exp(DT+\frac{\gamma_{13}MT}{\delta_{12}k_{12}})-1]}{\gamma_{22}\exp(-D\tau_2)}\}$,

(ii) $\tau_1<\frac{1}{D}(\ln\frac{(\alpha u_0+\alpha v_0)\gamma_{11}}{(k+M)(D+r_1M_3+r_{12}M)[\exp(DT+\frac{\gamma_{24}MT}{\delta_{21}k_{21}})-1]}-\mu M_4)$,
$\tau_2<\frac{1}{D}\ln\frac{(\alpha u_0+\alpha v_0)\gamma_{22}}{(k+M)(D+r_2M_3+r_{21}M)[\exp(DT+\frac{\gamma_{13}MT}{\delta_{12}k_{12}})-1]}$,

(iii) $T<\min\{\frac{1}{D+\frac{\gamma_{24}M}{\delta_{21}k_{21}}}\ln\frac{(\alpha u_0+\alpha v_0)\gamma_{11}\exp(-D\tau_1-\mu M_4)+(k+M)(D+r_1M_3+r_{12}M)}{(k+M)(D+r_1M_3+r_{12}M)}, \frac{1}{D+\frac{\gamma_{13}M}{\delta_{12}k_{12}}}\ln\frac{(\alpha u_0+\alpha v_0)\gamma_{22}\exp(-D\tau_2)+(k+M)(D+r_2M_3+r_{21}M)}{(k+M)(D+r_2M_3+r_{21}M)}\}$.

4 Numerical Simulation

1. Setting $T=2, u_0=1, v_0=1, c_0=1, p_0=1, D=0.2, \alpha=1, \gamma_1=0.3, \gamma_2=0.2, \gamma_3=0.14, \gamma_4=0.11, k_1=0.2, k_2=0.3, \delta_1=1, \delta_2=1, \mu=0.2, r_1=0.1, r_2=0.1, r_{12}=0.1, r_{21}=0.1, \tau_1=1, \tau_2=1$ so that the conditions of theorem 3.1 hold. Choose initial value $(0.01, 0.01, 0.01, 0.01, 0.01, 0.01)$, then we can see that the microorganism-free periodic solution $(\widetilde{u}(t), \widetilde{v}(t), 0, 0, \widetilde{c}(t), \widetilde{p}(t))$ of system (1.1) is global attractive(see Fig. 1).

2. Setting $T = 0.049, u_0 = 1, v_0 = 1, c_0 = 1, p_0 = 1, D = 7.4, \alpha = 1, \gamma_1 = 0.3, \gamma_2 = 0.2, \gamma_3 = 0.14, \gamma_4 = 0.11, k_1 = 0.2,\ k_2 = 0.3, \delta_1 = 1, \delta_2 = 1, \mu = 0.2, r_1 = 0.1, r_2 = 0.1, r_{12} = 0.1, r_{21} = 0.1, \tau_1 = 0.00000002, \tau_2 = 0.00000002$ so that the conditions of theorem 4.1 hold. Choose initial value $(0.1, 0.1, 1, 1, 0.01, 0.01)$, then we can see that the system (1.1) is permanent(see Fig. 2).

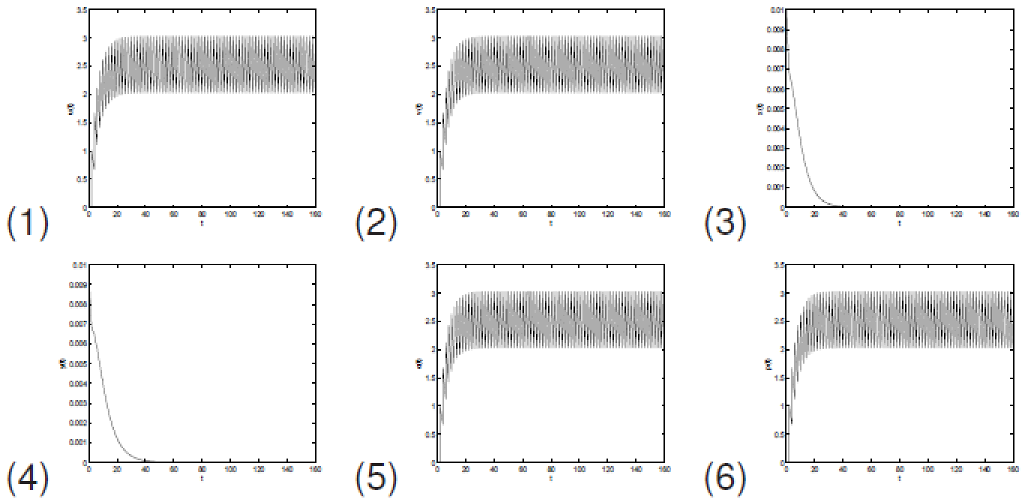

Figure 1: (1)-(6): The time series plot of $u(t)$, $v(t)$, $x(t)$, $y(t)$, $c(t)$, $p(t)$, the microorganism-free periodic solution is global attractive.

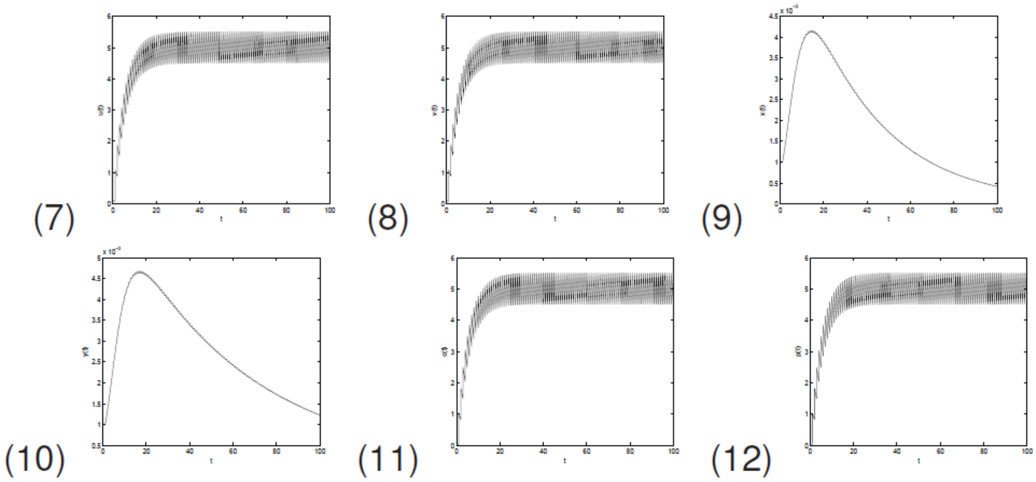

Fig 2. (7)-(12): The time series plot of u(t), v(t), x(t), y(t), c(t), p(t), system (1.1) is permanent.

5 Conclusion

In this paper, a two microorganisms and two nutrient chemostat competitive model with time delay and impulsive effect is considered. Besides, a polluted environment and an inhibitor was considered in this chemostat competitive model. By using the theorem of the impulsive equations and the theory of delay differential equations, we obtain some conclusions: (1) If the parameters satisfy the condition (3.1) or (3.2), then the system (1.1) will have microorganisms extinction periodic solution $(\widetilde{u}(t), \widetilde{v}(t), 0, 0, \widetilde{c}(t), \widetilde{p}(t))$, and which is global attractive. (2) If the parameters satisfy the condition $\Re > 1$, then the system (1.1) will be persistent.

The results of numerical simulation show that if the capacity of the microorganism uptake the nutrient is strengthened, the the capacity of the microorganism uptake the toxicant is weakened and the releasing amount of toxicant is lessened, the releasing period is elongated, the microorganism will persistent exist. So, we can protect the ecological balance via improve the habitus of microorganism and process bioconversion to toxicant.

Competing interests

The authors declare that no competing interests exist.

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