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π-Irreducible Mappings and K-Network of Infinite Compacts

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Abstract

In the paper the local density and the local weak density of topological spaces are investigated. It is proved that for a π-irreducible mapping f of a topological space X onto a topological space Y the followings hold: $d(X) = d(Y)$, $wd(X) = wd(Y)$, $ld(X) \leq ld(Y)$, $lwd(X) \leq lwd(Y)$. Moreover, it is showed that the functor of probability measures of finite supports P_n, the functor of the permutation degree SP^n_G and the functor $\exp_n$ preserve the cardinality of k-networks of infinite compacts.

Keywords: π-irreducible mapping; k-network; the local density; the local weak density; hyperspace; the space of the permutation degree.

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1 Introduction

Recall some definitions and propositions related to the work. Throughout this paper, all spaces are assumed to be infinite and all mappings are continuous and onto.

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Definition 1.1. A subset M of a topological space X is dense in X if $[M] = X$.

The density of a topological space is defined with following way: $d(X) = \min\{|M| : M\, is\, dense\, in\, X\}$. If $d(X) \leq \aleph_0$ for a space X we say that X is separable [1].

Definition 1.2. The weak density of a topological space X is the smallest cardinal number $\tau \geq \aleph_0$ such that there is a π-base in X coinciding with τ centered systems of open sets, i.e. there is a π-base $B = \bigcup\{B_\alpha : \alpha \in A\}$, where B_α is a centered system of open sets for each $\alpha \in A$ and $|A| = \tau$ [2].

The weak density of a topological space X is denoted by $wd(X)$. If $wd\,(X) \leq \aleph_0$ then we say that a topological space X is weakly separable.

Definition 1.3. We say that a topological space X is locally τ-dense at a point $x \in X$ if τ is the smallest cardinal number such that x has a τ-dense neighborhood in X.

The local density at a point x is denoted by $ld(x)$. The local density of a space X is defined as the supremum of all numbers $ld(x)$ for $x \in X$; this cardinal number is denoted by $ld(X)$. If $ld(X) \leq \aleph_0$ for a space X, we say that X is locally separable [3].

Definition 1.4. A topological space X is locally weakly τ dense at a point $x \in X$ if τ is the smallest cardinal number such that x has a neighborhood of weak density τ in X [4].

The weak density at a point x is denoted by $lwd(x)$. The local weak density of a topological space X is defined with following way: $lwd\,(X) = \sup\{lwd\,(x) : x \in X\}$. If $lwd(X) \leq \aleph_0$ for a space X, then we say that X is locally weakly separable [5].

2 Mappings Preserving the Local Density and the Local Weak Density

Definition 2.1. A continuous mapping $f : X \to Y$ of a space X onto a space Y is irreducible if $f(A) \neq Y$ for any proper closed subset A of the space X [3].

Definition 2.2. Let f be a mapping of X onto Y. We say that the map f is π-irreducible if for every proper closed subset $F \subset X$ its image $f(F)$ is not dense in Y [6].

It is obvious that a closed map is π-irreducible iff it is irreducible.

Definition 2.3. A mapping $f : X \to Y$ is an almost open mapping, if for each $y \in Y$ there exists $x \in f^{-1}(y)$ such that $f(U)$ is a neighborhood of y for each neighborhood U of x [7].

Definition 2.4. A mapping $f : X \to Y$ is called pseudo-open if for each $y \in Y$ and each neighborhood U of $f^{-1}(y)$ in X, $f(U)$ is a neighborhood of y in Y [8].

Theorem 2.1. *Let f be a continuous map of X onto Y. Then following statements are equivalent:*

1) f is π-irreducible;
2) for every π-base β of X and for every $B \in \beta$ the f-image of its compliment, $f(X \backslash B)$, is not dense in Y;
3) there is a π-base β of X such that for every $B \in \beta$ the f-image $f(X \backslash B)$ is not dense in Y;
4) for every π-base γ of Y the family $\{f^{-1}(C) : C \in \gamma\}$ is a π-base of X;
5) there is a π-base γ of Y such that the family $\{f^{-1}(C) : C \in \gamma\}$ is a π-base of X [6].

Theorem 2.2. *If $d(X) \leq \tau$ and $f : X \to Y$ is a continuous mapping of X onto Y, then $d(Y) \leq \tau$ [1].*

Proposition 2.1. *If $d(X) \leq \tau$ and G is an arbitrary non-empty open subset of the space X, then $d(G) \leq \tau$ [3].*

Theorem 2.3. *Let f be a π-irreducible mapping of X onto Y. Then $wd(X) = wd(Y)$.*

Proof. It is well known that the weak density is preserved under continuous mappings. Therefore $wd(Y) \leq wd(X)$. Let us now show $wd(X) \leq wd(Y)$. Let $wd(Y) = \tau$. This means that there is a π-base $\beta = \cup\{\beta_\alpha : \alpha \in A\}$ in Y, where $|A| \leq \tau$ and $\beta_\alpha = \{U_s^\alpha : s \in A_\alpha\}$ is a centered system of open sets in Y for every $\alpha \in A$. Then it is clear that $f^{-1}(\beta_\alpha) = \{f^{-1}(U_s^\alpha) : s \in A_\alpha\}$ is a centered system of open sets in X. This implies that the system $f^{-1}(\beta) = \cup\{f^{-1}(\beta_\alpha) : \alpha \in A\}$ coincides with τ centered systems of open sets. On the other hand, by theorem 2.1 $f^{-1}(\beta)$ is a π-base in X. Therefore, we obtain $wd(X) \leq \tau$. Theorem 2.3 is proved.

Theorem 2.4. *Let f be a π-irreducible mapping of X onto Y. Then $d(X) = d(Y)$.*

Proof. It is clear that $d(Y) \leq d(X)$. Now we shall show that $d(X) \leq d(Y)$. Let $d(Y) = \tau$. Then there is a dense subset $M = \{y_\alpha : \alpha \in A\}$ of Y such that $\overline{A} = \tau$. Let us choose a point x_α from each set $f^{-1}(y_\alpha)$ and form the set $M_1 = \{x_\alpha : \alpha \in A\}$. It is clear that $|M_1| = \tau$. Consider an arbitrary nonempty proper open subset $U \subset X$. Then clearly $X\backslash U$ is a proper closed subset of X. Since f is π-irreducible, we see that $f(X\backslash U)$ is not dense in Y. This implies that there is a nonempty open set $V \subset Y$ such that $V \subset Y\backslash f(X\backslash U)$. Since M is dense in Y, we have $y_\alpha \in M \cap V$ for some $y_\alpha \in M$. Then $f^{-1}(y_\alpha) \subset f^{-1}(V) \subset f^{-1}(Y\backslash f(X\backslash U)) = X\backslash f^{-1}(f(X\backslash U)) \subset X\backslash(X\backslash U) = U$. Hence $x_\alpha \in f^{-1}(y_\alpha) \subset U$ for some $x_\alpha \in M_1$. This means that M_1 is dense in X. Therefore $d(X) \leq \tau = d(Y)$. Theorem 2.4 is proved.

Theorem 2.5. *Let f be a π-irreducible mapping of X onto Y. Then $ld(X) \leq ld(Y)$.*

Proof. Let $ld(Y) = \tau$. Take an arbitrary point $x \in X$. Then $f(x) = y \in Y$. Since $ld(Y) = \tau$, there is a neighborhood Oy of y in Y such that $d(O_y) \leq \tau$. Note that $f^{-1}(Oy)$ is an open neighborhood of x. Let $M = \{y_\alpha : \alpha \in A\}$ be a dense subset of Oy with $|A| \leq \tau$. Let us choose a point x_α from each set $f^{-1}(y_\alpha)$ and form the set $M_1 = \{x_\alpha : \alpha \in A\}$. It is clear that $|M_1| \leq \tau$. We shall show that M_1 is dense in $f^{-1}(Oy)$. Consider an arbitrary nonempty open subset G of $f^{-1}(Oy)$. G is open in X as an open subset of the open subspace $f^{-1}(Oy)$. Since f is π-irreducible, $f(X\backslash G)$ is not dense in Y. Then there is a nonempty open set V in Y such that $V \cap f(X\backslash G) = \varnothing$. Hence $f^{-1}(V) \cap f^{-1}(f(X\backslash G)) = \varnothing$ and, a fortiori, $f^{-1}(V) \cap (X\backslash G) = \varnothing$. This implies $f^{-1}(V) \subset G$ and we have $V \subset f(G) \subset Oy$. On the other hand, $y_\alpha \in V$ for some $y_\alpha \in M$, since M is dense in Oy. Therefore $x_\alpha \in f^{-1}(y_\alpha) \subset f^{-1}(V) \subset G$ for $x_\alpha \in M_1$. This means that M_1 is dense in $f^{-1}(Oy)$. This implies $ld(x) \leq \tau$. We have chosen the point x arbitrarily, therefore $ld(X) \leq \tau$. Theorem 2.5 is proved.

Theorem 2.6. *Let f be a π-irreducible mapping of X onto Y. Then $lwd(X) \leq lwd(Y)$.*

Proof. Let $lwd(Y) = \tau$. Take an arbitrary point $x \in X$, then $f(x) \in Y$. Since $lwd(Y) = \tau$, there exists a neighborhood $Of(x)$ of the point $f(x)$ in Y such that $wd(Of(x)) \leq \tau$. This means that there is a π-base $\beta = \cup\{\beta_\alpha : \alpha \in A\}$ in $Of(x)$, where $|A| \leq \tau$ and $\beta_\alpha = \{U_s^\alpha : s \in A_\alpha\}$ is a centered system of open sets in $Of(x)$ for every $\alpha \in A$. Then it is clear that $f^{-1}(\beta_\alpha) = \{f^{-1}(U_s^\alpha) : s \in A_\alpha\}$ is a centered system of open sets in the neighborhood $f^{-1}(Of(x))$ of x. This implies that the system $f^{-1}(\beta) = \cup\{f^{-1}(\beta_\alpha) : \alpha \in A\}$ coincides with τ centered systems of open sets in $f^{-1}(Of(x))$. For completing the proof of the theorem it is sufficient to show that $f^{-1}(\beta)$ is a π-base in $f^{-1}(Of(x))$. Let G be a nonempty open subset of $f^{-1}(Of(x))$. Since f is π-irreducible, the set $f(X\backslash G)$ is not dense in Y. Then there is a nonempty open subset $V$of the space Y such that $V \cap f(X\backslash G) = \varnothing$. As it was noticed in the proof of theorem 2.3, we have $V \subset f(G) \subset Of(x)$. Since β is a π-base in $Of(x)$, we have $U_s^\alpha \subset V$ for some $U_s^\alpha \in \beta$. This implies $f^{-1}(U_s^\alpha) \subset f^{-1}(V) \subset G$ for $f^{-1}(U_s^\alpha) \in f^{-1}(\beta)$. This means that $f^{-1}(\beta)$ is a π-base in $f^{-1}(Of(x))$. Theorem 2.6 is proved.

Theorem 2.7. *Let f be an almost open mapping of X onto Y. Then 1) $ld(Y) \leq ld(X)$; 2) $lwd(Y) \leq lwd(X)$.*

Proof. 1) Let $ld(X) = \tau$. For every $y \in Y$ there is $x \in f^{-1}(y)$ such that $f(U)$ is a neighborhood of y in Y for an arbitrary neighborhood U of x. Since $ld(X) = \tau$, there is a neighborhood Ox of x such that $d(Ox) \leq \tau$. By theorem 2.2 we have $d(f(Ox)) \leq \tau$. On the other hand, $f(Ox)$ is a neighborhood of y in Y. So, we have found a neighborhood of density τ for arbitrarily taken point y. This implies $ld(Y) \leq \tau$. 1) is proved. The proof of 2) is the same as the proof of 1). Therefore, we omit it. Theorem 2.7 is proved.

Theorem 2.8. *Let $f : X \to Y$ be a pseudo-open compact mapping. Then 1) $ld(Y) \leq ld(X)$; 1) $lwd(Y) \leq lwd(X)$.*

Proof. 1) We shall prove $ld(Y) \leq ld(X)$. Let $ld(X) = \tau$. Let us take an arbitrary point $y \in Y$. Then the set $f^{-1}(y) \subset X$ is compact in X. For every point $x \in f^{-1}(y)$ there exists a neighborhood Ox of x such that $d(Ox) \leq \tau$. The family of all these neighborhoods covers the set $f^{-1}(y)$. Since $f^{-1}(y)$ is compact, there is a finite sequence $Ox_1, Ox_2, ..., Ox_n$ of open sets such that $f^{-1}(y) \subset \bigcup_{i=1}^{n} Ox_i$ and $d(Ox_i) \leq \tau$ for each $i = 1, 2, ..., n$. Put $G = \bigcup_{i=1}^{n} Ox_i$. Then we obtain $d(G) \leq \tau$. Since f is pseudo-open and $f^{-1}(y) \subset G$, we see that $y \in \mathrm{int}(f(G)) = Oy$. Then by theorem 2.2 and proposition 2.1 we have $d(Oy) \leq \tau$. We have found the neighborhood Oy of density $\leq \tau$ for arbitrarily chosen point $y \in Y$. Therefore $ld(Y) \leq \tau$. The inequality $ld(Y) \leq ld(X)$ is proved. The proof of the inequality $lwd(Y) \leq lwd(X)$ is the same as the proof of 1), therefore we omit it. Theorem 2.8 is proved.

3 k-Networks of Infinite Compacts

Let X be a T_1-space. The collection of all nonempty closed subsets of X we denote by $\exp X$. The family B of all sets in the form $O\langle U_1, ..., U_n\rangle = \Big\{ F : \ F \in \exp X,\ F \subset \bigcup_{i=1}^{n} U_i,\ F \cap U_i \neq \varnothing,$ $i = 1,\ 2,\ ...,\ n$, where $U_1, ..., U_n$ is a sequence of open sets of X, generates the topology on the set $\exp X$. This topology is called the Vietoris topology. The $\exp X$ with the Vietoris topology is called the exponential space or the hyperspace of X [1].

Denote by $\exp_n X$ the set of all closed subsets of X cardinality of that is not greater than the cardinal number n, i.e. $\exp_n X = \{\, F \in \exp X : \,|F| \leq n \,\}$.

Let X be a compact space. By $C(X)$ denote the set of all continuous maps $\phi : X \to R$ with the usual sup-norm $\|\phi\| = \sup\{|\phi(x)| : x \in X\}$. A continuous functional $\mu : C(X) \to R$ is called a measure on the compact X. A measure is positive (notation $\mu \geq 0$), if $\mu(\phi) \geq 0$ for any $\phi \geq 0$. A measure is normed, if $\|\mu\| = 1$. A positive normed measure is called a probability measure. A space consisting of all probability measures, is denoted by $P(X)$. A neighborhood base at a point $\mu \in P(X)$ consists of all the sets in the form $O(\mu; \phi_1, \phi_2, ..., \phi_k; \varepsilon) = \{\nu \in P(X) : |\mu(\phi_i) - \nu(\phi_i)| < \varepsilon,\ i = 1, 2, ..., k\}$ where $\phi_1, \phi_2, ..., \phi_k \in C(X)$ and $\varepsilon > 0$.

A support $\mathrm{supp}(\mu)$ of a measure $\mu \in P(X)$ is the smallest closed subset $F \subset X$ such that $\mu(F) = \mu(X)$. For a compact X and a natural number n put $P_n(X) = \{\mu \in P(X) : |\mathrm{supp}(\mu)| \leq n\}$ and $P_\omega(X) = \cup\{P_n(X) : n = 1,\ 2, ...\}$. It is easy to see that $P_\omega(X)$ is dense in $P(X)$ [1].

A permutation group X is the group of all permutations (i.e. one-one and onto mappings $X \to X$).

A permutation group of a set X is usually denoted by $S(X)$. If $X = \{1, 2, \ldots, n\}$, $S(X)$ is denoted by S_n, as well [9].

Let X^n be the n th power of a compact X. The permutation group S_n of all permutations, acts on the n th power X^n as permutation of coordinates. The set of all orbits of this action with quotient topology we denote by SP^nX. Thus, points of the space SP^nX are finite subsets (equivalence classes) of the product X^n. Thus two points $(x_1, x_2, \ldots, x_n)$, $(y_1, y_2, \ldots, y_n) \in X^n$ are considered to be equivalent if there is a permutation $\sigma \in S_n$ such that $y_i = x_{\sigma(i)}$. The space SP^nX is called the n-permutation degree of a spaces X [9]. Equivalence relations by which we obtained spaces SP^nX and $\exp_n X$, are called the symmetric and hypersymmetric equivalence relations, respectively. Any symmetrically equivalent points of X^n are hypersymmetrically equivalent. But the inverse is not correct. So, points (x, x, y), $(x, y, y) \in X^3$ are hypersimmetrically equivalent, but not symmetrically equivalent.

The concept of a permutation degree has generalizations. Let G be any subgroup of the group S_n. Then it also acts on X^n as group of permutations of coordinates. Consequently, it generates a G-symmetric equivalence relation on X^n. The quotient space of the product X^n under the G - symmetric equivalence relation, is called G-permutation degree of the space X and is denoted by SP^n_GX. An operation SP^n_G is also the covariant functor in the category of compacts and is said to be a functor of G-permutation degree. If $G = S_n$ then $SP^n_G = SP^n$. If the group G consists only of unique element then $SP^n_GX = X^n$. Moreover, if $G_1 \subset G_2$ for subgroups G_1, G_2 of the permutation group S_n then we get a sequence of the factorization of functors:

$$X^n \to SP^n_{G_1} \to SP^n_{G_2} \to SP^n \to \exp_n \ (3.1)$$

Definition 3.1. Let P be a family of subsets of a space X and $\tau(X)$ is the topology on X. P is called a k - network if whenever K is a compact subset of X and $K \subset U \in \tau(X)$, there is a finite subfamily $P' \subset P$ such that $K \subset \bigcup P' \subset U$ [10].

Theorem 3.1. *If $f : X \to Y$ is a perfect mapping, then for every compact subspace $Z \subset Y$ its inverse image $f^{-1}(Z)$ is compact [6].*

Proposition 3.1. *Let $f : X \to Y$ be a perfect mapping of a topological space X onto a topological space Y. If X has a k-network of cardinality $\tau \geq \aleph_0$, then Y has a k-network of cardinality $\leq \tau$.*

Proof. Let $f : X \to Y$ be a perfect onto map and let $P = \{E_\alpha : \ \alpha \in A\}$ be a k- network of cardinality $\tau \geq \aleph_0$ in X. Let us show that the family $f(P) = P_1 = \{f(E_\alpha) : \ \alpha \in A\}$ is a k - network of cardinality τ for Y. It is clear that $|P_1| \leq \tau$. Let K be an arbitrary compact subspace of Y and let U be an arbitrary neighborhood of K. Then $f^{-1}(U)$ is an open set in X, containing the compact $f^{-1}(K)$. Since $f^{-1}(K)$ is compact, there is a finite subfamily $P' = \{E_{\alpha_1}, E_{\alpha_2}, \ldots, E_{\alpha_n}\}$ of such that $f^{-1}(K) \subseteq \bigcup\limits_{i=1}^{n} E_{\alpha_i} \subseteq f^{-1}(U)$. According to the equality $f(\bigcup\{E_\alpha : i = 1, 2, ..., n\}) = \bigcup\{f(E_\alpha), i = 1, 2, ..., n\}$ we see that $K \subset \bigcup\limits_{i=1}^{n} f(E_{\alpha_i}) \subset U$. Proposition 3.1 is proved.

Proposition 3.2. *Suppose that topological spaces X and Y have k-networks of cardinality $\tau \geq \aleph_0$, then their product $X \times Y$ has a k-network of cardinality $\leq \tau$.*

Proof. Let $P_1 = \{E_\alpha : \ \alpha \in A\}$ and $P_1 = \{M_\beta : \ \beta \in B\}$ be k-networks in X and Y, respectively. We show that $P_1 \times P_2 = \{E_\alpha \times M_\beta : \ \alpha \in A, \ \beta \in B\}$ is a k-network of cardinality $\leq \tau$ in $X \times Y$. Let $K \subset X \times Y$ be an arbitrary compact and let G - be its arbitrary neighborhood in $X \times Y$. Then $\pi_1(K) = K_1$ and $\pi_2(K) = K_2$ are compacts in X and Y, respectively. Furthermore, $\pi_1(G) = G_1$ and $\pi_2(G) = G_2$ are neighborhoods of compacts K_1 and K_2, respectively. Since

P_1 and P_2 are k-networks in X and Y, respectively, there exist elements $E_{\alpha_1}, E_{\alpha_2}, \ldots, E_{\alpha_n} \in P_1$, $M_{\alpha_1}, M_{\alpha_2}, \ldots, M_{\alpha_n} \in P_2$, such that $K_1 \subset \bigcup\limits_{i=1}^{k} E_{\alpha_i} \subset G_1$ and $K_2 \subset \bigcup\limits_{j=1}^{s} E_{\alpha_j} \subset G_2$ in X and Y, respectively. Then $K_1 \subset \bigcup\limits_{i,=1}^{k,\,s} E_{\alpha_i} \times M_{\beta_j} \subset G$. Therefore, $P_1 \times P_2$ is a k-network cardinality $\leq \tau$ in $X \times Y$. Proposition 3.2 is proved.

Corollary 3.2. *If $P = \{E_\alpha : \alpha \in A\}$ is a k-network of cardinality $\tau \geq \aleph_0$ in X, then $P^n = \{E_{\alpha_1} \times E_{\alpha_2} \times \ \ldots \times E_{\alpha_n} : \ E_{\alpha_i} \in P, \ i = 1, 2, \ldots, n\}$ is a k-network of cardinality $\leq \tau$ in X^n.*

Corollary 3.3. *Suppose that topological space X have k-network of cardinality $\tau \geq \aleph_0$, then the space X^n has a k-network of cardinality $\geq \tau$.*

Theorem 3.4. *Let X be an infinite compact T_2-space with a k-network of cardinality $\tau \geq \aleph_0$ and G be an arbitrary subgroup of the group S_n. If G_1 and G_2 are subgroups of the permutation group S_n that $G_1 \subset G_2$, then spaces $\prod^n(X)$, $SP^n_{G_1}(X)$, $SP^n_{G_2}(X)$, $SP^n(X)$, $\exp_n X$, $P_n(X)$ have a k-network of cardinality $\leq \tau$.*

Proof. Let X be an infinite compact T_2-space with a k-network of cardinality $\tau \geq \aleph_0$. Then by corollary 3.1, the compact X^n has a k-network of cardinality $\leq \tau$. It is known that $SP^n(X)$ is a quotient space of X^n. Since a quotient mapping is "onto", by proposition 3.1 and equalities (3.1), we see that each of the spaces $SP^n_{G_1}(X)$, $SP^n_{G_2}(X)$, $SP^n(X)$, $exp_n X$ has a k-network of cardinality $\leq \tau$.

In [11] it is shown that $P_n(X)$ can be represented as a continuous image of the space $X \times \sigma^{n-1}$, where σ^{n-1} is the (n-1)-dimensional simplex. The mapping $\pi : X \times \sigma^{n-1} \to P_n(X)$ is defined with the formula $\pi(x_1, ..., x_n, m_1, ..., m_n) = \sum\limits_{i=1}^{n} m_i \delta_{x_i}$, where $(m_1, ...m_n) \in \sigma^{n-1}$, $\sum\limits_{i=1}^{n} m_i = 1$ and $m_i \geq 0$ for each $i \in N$, δ_{x_i} is Dirak's measure at point x_i, respectively. The mapping π is perfect, since π is continuous mapping defined on compact $X \times \sigma^{n-1}$. Therefore, by proposition 3.1, we see that the space $P_n(X)$ has a k-network of cardinality $\leq \tau$. Theorem 3.4 is proved.

Corollary 3.5. Functors $\prod^n$, $SP^n_{G_1}$, $SP^n_{G_2}$, SP^n, exp_n, P_n preserve k-network of infinite compacts.

4 Conclusions

In the paper k-networks, the density, the weak density, the local density and the local weak density of topological spaces are investigated. In section 2 it is proved that π-irreducible mappings preserve the density and the weak density. Besides, it is shown that the local density (the local weak density) of the inverse image of a topological space under π-irreducible mapping is not greater than the local density (respectively, the local weak density) of the space. In section 3 k-networks of infinite compacts are considered. The main result in section 3 is that functors finite product, the permutation group, exponential functor and the functor of probability measures preserve the cardinality of k-networks for infinite compacts.

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Competing Interests

The authors declare that no competing interests exist.

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