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# Isothermal and Adiabatic Expansion Based Trilateral Cycles

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Author RFG designed the study, wrote the protocol, the first draft of the manuscript and managed literature searches. Author BFS managed the analyses of the study and literature searches. Both authors read and approved the final manuscript.*

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## ABSTRACT

This research work deals with non condensing mode trilateral thermal cycles characterised by the conversion of heat into mechanical work undergoing optionally isothermal and adiabatic closed process based path functions. The proposed trilateral cycles are also characterised by its ability to operate at high efficiency at relative low temperatures when compared with the Carnot factor.
An analysis of the proposed cycles is carried out and results were compared with that of a Carnot cycle operating under the same ratio of temperatures. It is determined that into a range of relative low operating temperatures high thermal efficiency is achieved, reaching 41.1% for helium when Carnot Factor is 9.1% under a ratio of temperatures of 300/330 K.

*Keywords: Carnot factor; adiabatic expansion; isothermal expansion; load-dependent path functions; thermal efficiency; trilateral cycle.*

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**Nomenclature**

| | |
|---|---|
| $\gamma$ | *adiabatic expansion coefficient* |
| $\eta$ | *thermal efficiency* |
| $\eta_I$ | *thermal efficiency (isothermal based TC)* |
| $\eta_A$ | *thermal efficiency (adiabatic based TC)* |
| *Cp* | *specific heat capacity at constant pressure (kJ/kg-K)* |
| *Cv* | *specific heat capacity at constant volume (kJ/kg-K)* |
| *R* | *Perfect gases constant (kJ/kg-K)* |
| *p* | *pressure (bar)* |
| $qi=q_{23}$ | *total specific input heat (kJ/kg)* |
| $q_o=q_{21}$ | *specific rejected heat (kJ/kg)* |
| *s* | *specific entropy (kJ/kg-K)* |
| *T* | *temperature (K)* |
| $W_I$ | *specific work (isothermal TC) (kJ/kg)* |
| $W_A$ | *specific work (adiabatic TC) (kJ/kg* |

**Acronyms**

| | |
|---|---|
| *C* | *Constant* |
| *CF* | *Carnot factor, Carnot efficiency* |
| *ORC* | *Organic Rankine Cycle* |
| *NORC* | *Non-organic Rankine Cycle* |
| *TC* | *Trilateral thermal cycle* |
| *WF* | *Working fluid* |

## 1. INTRODUCTION

The conventional thermal cycles under use so far are based on the Carnot engine, that are quadrilateral cycles in which ideally heat is absorbed at high temperature (top temperature) to be expanded while performing mechanical work undergoing temperature decrease from the top temperature to approach the bottom temperature under a quasi isentropic transformation. In Fig. 1 (a), a simple classification of heat engines based cycles of this type is depicted.

The power cycles that obey such thermodynamic model are classified in two main groups on the basis of the nature of the working fluid according to the Fig. 1 (a): gas power cycles and vapour power cycle. The difference between the two groups is that in the first case the working fluid is gaseous and does not experiment any phase change, while for the second group, there is a liquid-vapour phase change process of the working fluid within the cycle. Some studies regarding TCs have recently appeared in scientific literature. Thus, in the low temperature range, bottoming ORCs (organic Rankine cycles) constitute an interesting alternative to exploit the heat to power conversion cycles, having shown good thermodynamic performance for bottoming cycles [1-2]. The interest in organic WFs for residual heat applications with low temperature Rankine cycles is an old technique that has been proposed for different applications such as renewable energy and low temperature heat recovery [3-7]. Recent advances concerning low grade heat sources applied to ORCs include the works of Wang and colleagues [8], who compared several WFs for low-temperature ORCs, concluding that R123 is the best choice for the temperature range of 100 to 180°C and R141b is the optimal working fluid when the temperature is higher than 180°C. In the same way, Jianqin and colleagues [9] proposed using an open steam power cycle for internal combustion (ICE) engine exhaust gas energy recovery. The authors concluded that the recovery efficiency of exhaust gas energy is mainly limited by exhaust gas temperature and ICE thermal efficiency can be improved by 6.3% at 6000 r/min. Hua and colleagues [10] presented an ORC system used in ICE exhaust heat recovery, and a techno-economic analysis based on various WFs. They recovered a significant amount of ICE exhaust heat, which represented about one-third of the energy generated from the fuel by the ORC system. Results showed that R141b, R123 and R245fa present the highest thermal efficiency ranged from 16.60% to 13.30% and net power values from 60 to 49 kJ/kg. Jiangfeng and colleagues [11] conducted a study on low-temperature ORC which examined the effects of key thermodynamic design parameters, including turbine inlet pressure, turbine inlet temperature, pinch temperature difference and approach temperature difference in HRVG (heat recovery vapour generator), on the net power output and surface areas of both the HRVG and the condenser using R123, R245fa and isobutene. The results showed that turbine inlet pressure, turbine inlet temperature, pinch temperature difference and approach temperature difference had significant effects on the net power output and surface areas of both the HRVG and the condenser.

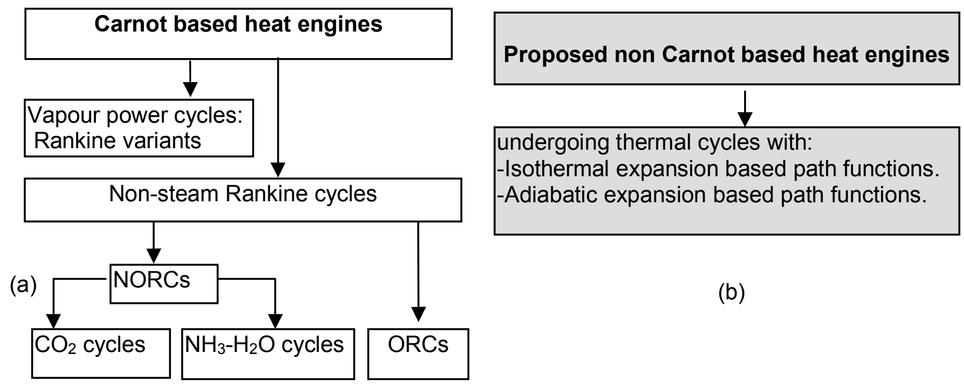



**Fig. 1. Classification of Carnot and non Carnot based thermal engines including quadrilateral and trilateral thermal cycles (TCs)**

The study carried out by Dongxiang and colleagues [12] proposed an ideal ORC to analyse the influence in working fluid properties on thermal efficiency. The optimal operating conditions and exergy destruction for various heat source temperatures were also evaluated by means of pinch point analysis and exergy analysis. The results showed that different WFs have little impact on the optimal operating condition of ORC and selection of working fluid reasonably based on heat source temperature will help to optimise ORC performance.

During the last decade, growing interest has been observed in utilizing low and medium temperature heat sources, mainly due to its availability from oceans, solar, geothermal and industrial residual heat sources. Considering the field of heat to power conversion applications, conventionally ORCs (organic Rankine cycles) and Kalina cycles have been applied. And recently TCs (trilateral cycles) [13-15] are being implemented. Thus, for example, the TC performance has been researched for ammonia-water as working fluid in [14] and ORCs and TCs have been thermodynamically compared in [15]. The author presents a comparison of optimized systems with trilateral cycles with water as working fluid and optimized organic Rankine cycles, where the optimization criterion is the exergy efficiency for power production being the ratio of the net power output to the incoming exergy flow of the heat carrier. The author claims that the exergy efficiency for power production is larger by 14% - 29% for the trilateral cycle than for the ORC.

Whilst ORC and Kalina cycles are used already in existing power plants, the TC, although theoretically mature, it is still under technical development [13]. The components to implement a conventional TC are similar to those in the ORC system except that the working fluid at the entrance of the TC expander (turbine) is a saturated liquid. AS consequence, the state of the working fluid at the expander exit is a two-phase mixture. As the thermodynamic mean temperature at which heat is received is comparatively lower for the TC, the thermal efficiency for this cycle is lower than that for the ORC, for the same temperature limits. The performance of a trilateral cycle has been recently analysed in [16]. Also investigates and compares it with those for the ORC (organic Rankine cycle) and the Kalina cycle, focused on thermodynamics and thermo economics aspects. The results for the TC indicate that an increase in the expander inlet temperature leads to an increase in net output power and a decrease in product cost for this power plant, whereas this is not the case for the ORC system.

Quasi-isothermal compression based reciprocating engines as well as Quasi-isothermal expansion based power cycle engines have been recently developed. In this way, M.W. Coney et al. [17] presented the analysis of a novel concept for a high efficiency reciprocating internal combustion engine called as the isoengine and its cycle, where the maximum net electrical plant efficiency has been predicted to approach about 60% on diesel fuel and 58% on natural gas. They concluded that the key to the high electrical efficiency is the quasi-isothermal compression of the combustion air in cylinders, saving compression work and allowing the recovery of waste heat back into the cycle, mainly from the exhaust gas by means of a recuperator. On the other hand, Opubo N et al. [18], investigated some vapour power cycles for quasi-isothermal expansion instead of adiabatic expansion, taking advantage of the fact that quasi-isothermal expansion has the advantage of bringing the cycle efficiency closer to the ideal Carnot efficiency, with the drawback of requiring heat to be transferred to the working fluid as it expands. The authors claimed that the comparison the specific work output to more standard Stirling engines using gas is higher.

The research effort in this work is focused on trilateral thermal cycles operating under quasi-isothermal expansion instead of adiabatic expansion. Throughout this contribution it will be shown that for low relative operating temperatures the thermal efficiency exceeds the Carnot factor, which makes this thermal cycle very suitable for exploiting the heat to power conversion cycles with low grade residual heat.

However, in the present work the closed isothermal path functions carried out in the TCs, are appropriated to operate at relative low temperatures, being based on a completely different thermal cycle concept in comparison with the conventional Carnot based thermal cycles, not only structurally but also in terms of Carnot factor (CF) constraints, which can be surpassed in some particular operating conditions [19].

Thus, the objective of the study is to efficiently convert low temperature heat mainly from ocean thermal, solar thermal and low grade heat from industrial residual power sources by means of the proposed TCs into electric energy. As it has been indicated, the proposed conversion method is based on a different thermodynamic cycle: a non condensing mode closed processes based TC with or without regeneration. With this contribution, most of the low grade rejected heat may be converted into mechanical work at acceptable thermal efficiency by applying a TC whose thermal efficiency is not restricted by the CF such as mentioned in [19].

## 2. ANALYSIS OF THE TC UNDERGOING ADIABATIC AND ISOTHERMAL PATH FUNCTION

This section deals with the ideal transformations in terms of conversion heat into mechanical work that could be carried out in the presented closed processes based thermal cycles. In this way, the fact of introducing the concept of displacement work undergoes the movement of the piston in a cylinder from position i to position i+1, while volume changes from $V_i$ to $V_{i+1}$, causing an amount of mechanical work W done by the system which is given as

$$W_{i-i+1} = \int_{V_1}^{V_2} p \cdot dV$$

In order to analyse the Adiabatic and Isothermal ideal cases, let's consider the displacement of the piston under different path functions as shown in Fig.2, for the following processes denoted as

-Adiabatic process ($pV^{\gamma} = C$)

-Isothermal process (constant temperature)

Fig 2, depicts the mechanical work performed under different thermodynamic transformations when evolving from the state point i to the state point i+1, denoted as the state point 1 and state point 2 respectively. (a), Isothermal, (b) adiabatic.

### 2.1 The Mechanical Work under an Isothermal Path Function. Fig. 2(a)

Assuming that $W_{1-2} = \int_{V_1}^{V_2} p \cdot dV$ (1)

and

$$p \cdot V = p_1 \cdot V_1 = p_2 \cdot V_2 \rightarrow \frac{V_1}{V_2} = \frac{p_2}{p_1} \rightarrow p = \frac{p_1 \cdot V_1}{V} \quad (2)$$

The mechanical work delivered under an isothermal process is

$$W_{1-2} = p_1 \cdot V_1 \int_{V_1}^{V_2} \frac{dV}{V} = p_1 \cdot V_1 \cdot Ln \frac{V_2}{V_1} = p_1 \cdot V_1 \cdot \ln \frac{p_1}{p_2} = R \cdot T_1 \cdot Ln \frac{p_1}{p_2} \quad (3)$$

### 2.2 The Mechanical Work under an Adiabatic Path Function. Fig. 2(b)

The adiabatic process is represented by

$$(p \cdot V^{\gamma} = C)$$

where $\delta Q$ or $dQ = 0$ and the adiabatic exponent $\gamma = \frac{Cp}{Cv}$

Since $\delta Q = dU + dW = 0$ ,

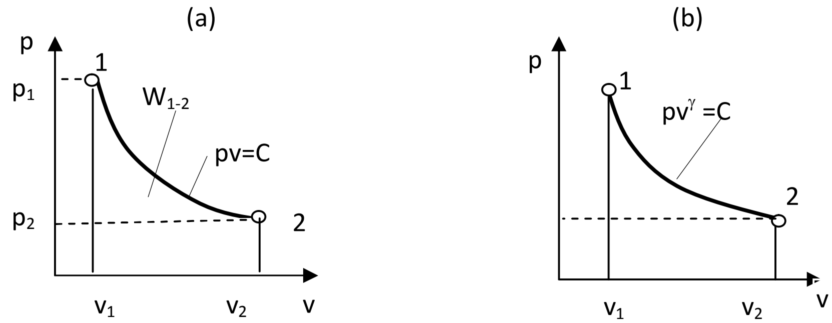



**Fig. 2. Mechanical work performed under adiabatic and isothermal thermodynamic transformations: (a), isothermal path function. (b), adiabatic path function**

Then

$$dW = dU = -Cv \cdot dT$$
$$dW = p \cdot dV \tag{4}$$

With

$$[p_1 \cdot V_1^{\gamma} = p_2 \cdot V_2^{\gamma} = C]$$

The mechanical work delivered under an adiabatic process is

$$W_{1-2} = \int_{V_1}^{V_2} \frac{C}{V^{\gamma}} dV = C \int_{V_1}^{V_2} V^{-\gamma} dV = C \frac{V^{-\gamma+1}}{-\gamma+1} = \tag{5}$$
$$= \frac{C}{1-\gamma}[V_2^{1-\gamma} - V_1^{1-\gamma}] = \frac{p_2 V_2^{\gamma} V_2^{1-\gamma} - p_1 V_1^{\gamma} V_1^{1-\gamma}}{1-\gamma}$$

$$W_{1-2} = \frac{p_2 V_2 - p_1 V_1}{1-\gamma} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} \tag{6}$$

## 2.3 The Isothermal and Adiabatic Transformations in the TC

The proposed TC obey the processes included between the state points defined in Table 1. In this way, Fig. 3 shows the T-s and p-V diagrams of a feasible thermal engine undergoing two optional closed process based transformations for converting heat into mechanical work, where Fig. 3(a) and Fig. 3(b), depicts the isothermal transformation for the task of heat to work conversion and the Fig. 3(c) and Fig. 3(d) depicts the adiabatic transformation for the task of heat to work conversion.

Two different and optional engine structures to implement a TC composed by closed system based transformations are depicted in Fig 4. As shown there, it is depicted a double cylinder implemented under a pair of bellows type actuators operating with helium as working fluids. The cycle status corresponds to the chamber 1 active and chamber 2 passive. Under these conditions,

## 2.4 TC Analysis Undergoing Adiabatic or Isothermal Path Functions

According to the information provided by the Table 1 and the Fig. 3, the transformations associated to each cycle are summarised as follows:

**Table 1. The TC showing the closed processes for the isothermal and adiabatic cases**

| (i)-(i+1) | TC undergoing Isothermal heat conversion path function | TC undergoing adiabatic heat conversion path function |
|---|---|---|
| (1)-(2) | Closed isochoric process | Closed isochoric process |
| (2)-(3) | Closed isothermal process | Closed adiabatic process |
| (3)-(1) | Closed isobaric process | Closed isobaric process |

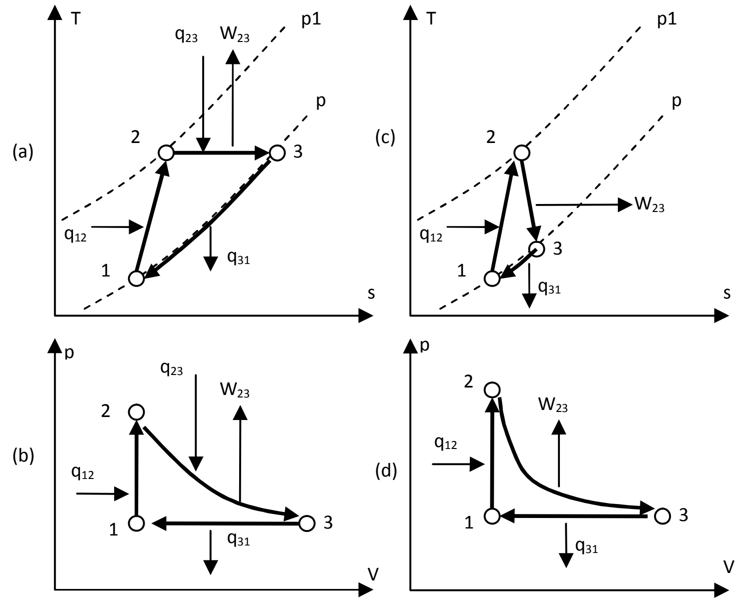
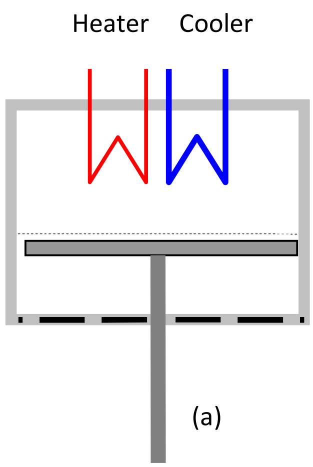


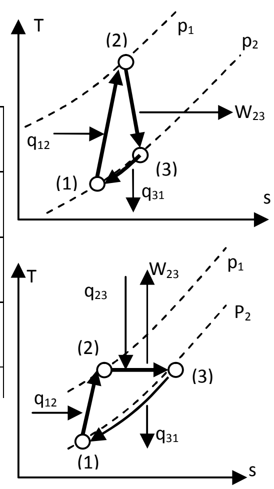

**Fig. 3. The T-s and p-V diagrams of the thermal engine undergoing two optional closed process based transformations for converting heat into mechanical work**




(a)

| points | process | X | Y |
|---|---|---|---|
| | **Adiabatic** TC | | |
| (1)-(2) | isochoric | 1 | 0 |
| (2)-(3) | **adiabatic** | 0 | 0 |
| (3)-(1) | isobaric | 0 | 1 |

(b)




(c)

**Fig. 4. A closed processes based TC implemented with a single effect cylinder**

### 2.4.1 The isothermal expansion based TC

Leg (1)-(2):

Correspond to a closed isochoric heating process. The amount of heat added from an external heat source at constant volume is

$$W_{12} = 0, \quad q_{12} = u_2 - u_1 = Cv \cdot (T_2 - T_1) \qquad (7)$$

Leg (2)-(3):

Correspond, to a closed isothermal heating process. Consequently the internal energy remains constant so that the total heat added is converted into mechanical works since ideally there are not internal energy changes

$$u_{23} = 0, \quad q_{23} = W_{23} = p_2 \cdot v_2 \cdot Ln(\frac{p_2}{p_1}) = R \cdot T_2 \cdot Ln(\frac{p_2}{p_1}) \qquad (8)$$

Leg (3)-(1):

Correspond to a closed isobaric process in which the working fluid is cooled. There is not mechanical work so that the rejected heat is coming from the internal energy according to the function

$$W_{31} = 0, \quad q_{31} = u_3 - u_1 = Cv \cdot (T_2 - T_1) = Cv \cdot (T_3 - T_1) \tag{9}$$

According to the expressions (7) - (9), the thermal efficiency of the TC undergoing a closed isothermal expansion is given as

$$\eta_I = 1 - \frac{q_o}{q_i} = 1 - \frac{q_{31}}{q_{12} + q_{23}} = 1 - \frac{q_{31}}{q_{13}} = \frac{W_{23}}{q_{13}} \tag{10}$$

Therefore the thermal efficiency of an isothermal expansion based TC can be expressed analytically, as

$$\eta_I = \frac{W_{23}}{q_{13}} = \frac{W_{23}}{(u_2 - u_1) + W_{23}} = \frac{W_{23}}{Cv \cdot (T_2 - T_1) + W_{23}} = \frac{R \cdot T_2 \cdot Ln\left[\frac{p_2}{p_1}\right]}{Cv \cdot (T_2 - T_1) + R \cdot T_2 \cdot Ln\left[\frac{p_2}{p_1}\right]} \tag{11}$$

Since ideally the heat added to the transformation (2)-(3) is converted into mechanical work, the thermal efficiency defined in (11) exhibits a strong dependence on the difference of temperatures ($T_2$-$T_1$). Apparently, the thermal efficiency tends to the unity when such temperature difference tends to zero. However, real gases behaviour differs from this model.

A useful relation between (11) and the CF can be stated according to the following considerations:

Assuming the $CF = 1 - \frac{T_1}{T_2} = \frac{T_2 - T_1}{T_2}$, and the isothermal TC efficiency as $\frac{W_{23}}{Cv \cdot (T_2 - T_1) + W_{23}}$ follows that

$$\eta_I = \frac{W_{23}}{W_{23} + T_2 \cdot Cv \cdot CF} \tag{12}$$

Equation (12) suggest to us that for any amount of mechanical work $W_{23}$ done, as $T_2$ increases and consequently CF increases, the isothermal TC efficiency $\eta_I$ decreases and vice versa.

#### 2.4.2 The adiabatic expansion based TC

Leg (1)-(2):

Correspond to a closed isochoric heating process. The amount of heat added from an external heat source at constant volume is

$$W_{12} = 0, \quad q_{12} = u_2 - u_1 = Cv \cdot (T_2 - T_1) \tag{13}$$

Leg (2)-(3):

Correspond, to a closed adiabatic process. Consequently, since there is not heat transfer from an external source, the internal energy is converted into mechanical work according to the general expression

$$q_{23} = 0, \quad u_2 - u_3 = W_{23} = \frac{p_2 \cdot v_2 - p_3 \cdot v_3}{\gamma - 1} \tag{14}$$

Leg (3)-(1): v

Correspond to a closed isobaric process in which the working fluid is cooled. There is not mechanical work so that the rejected heat is coming from the internal energy according to the function

$$W_{31} = 0, \quad q_{31} = u_3 - u_1 = Cv \cdot (T_3 - T_1) \tag{15}$$

According to the expressions (7) - (9), the thermal efficiency of the TC undergoing a closed isothermal expansion is given as

$$\eta_A = 1 - \frac{q_o}{q_i} = \frac{W_{23}}{q_{13}} \tag{16}$$

Therefore the thermal efficiency for an adiabatic expansion based TC can be expressed analytically, as

$$\eta_A = 1 - \frac{q_o}{q_i} = 1 - \frac{Cv \cdot (T_3 - T_1)}{Cv \cdot (T_2 - T_1)} = \frac{(T_2 - T_3)}{(T_2 - T_1)} = \frac{p_2 \cdot v_2 - p_3 \cdot v_3}{(\gamma - 1) \cdot Cv \cdot (T_2 - T_1)} = \frac{R \cdot (T_2 - T_3)}{(\gamma - 1) \cdot Cv \cdot (T_2 - T_1)} \tag{17}$$

A useful relation between (16) with the CF can be stated as follows:

Considering that $CF = 1 - \frac{T_1}{T_2} = \frac{T_2 - T_1}{T_2}$, and the adiabatic TC efficiency as $\eta_A = \frac{(T_2 - T_3)}{(T_2 - T_1)}$ follows that

$$CF \cdot \eta_A + \frac{T_3}{T_2} = 1 \quad (18)$$

Equation (18) indicates that for low top temperature $T_2$, the value of CF is also low which implies that as $T_2$ tends to decrease then $\eta_A$ tends to increase. The consequence of such conclusion undergoes the possibility of surpassing the value of the CF for low top cycle temperatures while satisfying Clausius and Kelvin-Planck statements or second law.

## 3. A CASE STUDY BASED ON THE TC COMPOSED BY CLOSED SYSTEMS BASED TRANSFORMATIONS

This section deals with the exploration and analysis of a case study in which a TC composed by closed systems based transformations is applied on the conversion of heat to mechanical work undergoing adiabatic or isothermal path functions.

The Fig. 4 represents a closed processes based TC implemented by means of a single effect cylinder in which (a) shows the single effect cylinder, (b), shows the processes corresponding to the state points, and (c), shows the corresponding T-s diagrams for adiabatic and isothermal expansions.

The TC shown In Fig 4 consists of a single effect cylinder-actuator which could operate optionally with helium, hydrogen, as well as other gases that remain at gaseous state at ambient temperature. The cylinder is equipped with heating and cooling facilities. The piston by means of the rod could drive a reciprocating hydraulic pump (not represented) which will drive a turbine by means of a hydraulic circuit. Although not analysed in this work and consequently not represented in Fig. 4, the possibility of regenerating some residual heat exits due to the fact that the working fluid temperature at the end of the expansion is yet useful. Consequently, the possibility of increasing the thermal efficiency by regeneration is a real fact. The thermal cycle depicted in Fig. 4 can be easily understood by observing the associated table as well as the T-s diagrams for a cycle with adiabatic expansion and a cycle with isothermal expansion.

In the associated table of Fig. 4 the ports X and Y means the heater and cooler heat flows. Thus X = 1 means that heat is entering to the cylinder, while Y = 1 means that heat is being rejected from the cylinder to the environment. X=Y=0 indicates no heat transfer activity.

According to the information provided by the table embedded in the Fig. 4, it follows that during the process carried out between the state points (1)-(2), heat is supplied at constant volume, so that the piston remains unmovable.

After the temperature of the working fluid reaches the desired value, then a transformation (2)-(3) is carried out. During this transformation an adiabatic expansion is performed in the case of adiabatic transformation which is carried out without heat addition or an isothermal expansion is performed if a controlled heat flow rate is provided to satisfy the isothermal expansion.

After expansion has finished in the state point (3), the piston initiates the way back towards the point (1) at constant pressure, while is being cooled, so that the cycle is completed.

## 4. RESULTS OF THE ANALYSED CASE STUDY

On the basis of the data shown in Table A1, with data from [20], the thermal efficiency and the specific work of the TC has been computed for several top temperatures and two different expansion path functions: adiabatic and isothermal. The results of this study are shown in Table 2. It is observed that very low difference of thermal efficiency between isothermal expansion based TC or adiabatic expansion based TC exist. According to the achieved results, within the range of top temperatures between 330 and 900 K, the thermal efficiency and specific work are depicted. It is shown that for top temperatures below 590 K (corresponding to a pressure slightly below 2 bar), CF is lower than thermal efficiency for helium as working fluid considered as a real gas. However, with regard to the specific work a significant difference exists. As shown in Fig. 5. it is depicted the thermal efficiency for the adiabatic and isothermal expansion based TC with helium as working fluid for a closed processes based TC implemented with a single effect cylinder depicted in Fig. 4.

The specific work for helium as working fluid is depicted in Fig. 6, also shown in Table 2. It is shown that the specific work is much higher for the isothermal expansion based TC than for the adiabatic expansion based TC. In fact there exists a significant difference. This suggests to us that the structure of the cylinder operating under isothermal expansion based TC, requires a smaller volumetric size than for the case of adiabatic expansion based TC to deliver the same power.

### 4.1 The Isothermal and Adiabatic Transformations undergoing Different WFs

In order to validate the helium as working fluid, some other working fluids have been studied, such as carbon dioxide, nitrogen and hydrogen. The mentioned working fluids have been studied taking into account its availability since the cost is not relevant due to the negligible cost of the working fluids consumption. However after the comparison analysis of the attained results, the selection of the best working fluid for a specific application according to a performance criterion consists of a deterministic task.

The TC has been computed for the different WFs: carbon dioxide, nitrogen, helium and hydrogen. The results of the cycle computation are shown in Table 3, where it is shown a comparison of the thermal efficiencies and the specific works for carbon dioxide, nitrogen, helium and hydrogen as working fluids undergoing isothermal and adiabatic path functions.

From the data attained as the results of the TC computation depicted in the Table 3, it can be highlighted that the fact of converting heat into work undergoing adiabatic of isothermal path functions is not relevant in terms of thermal efficiency since these parameters are very similar for nitrogen, helium and hydrogen, although in the case of carbon dioxide, some relevant difference exists. However, with regard to the specific work, there exist relevant differences for the four considered working fluids.

When considering the characteristics of any analysed working fluid, great differences exist. Thus, for instance, thermal efficiency increases as function of the WF order: carbon dioxide, nitrogen, hydrogen and helium.

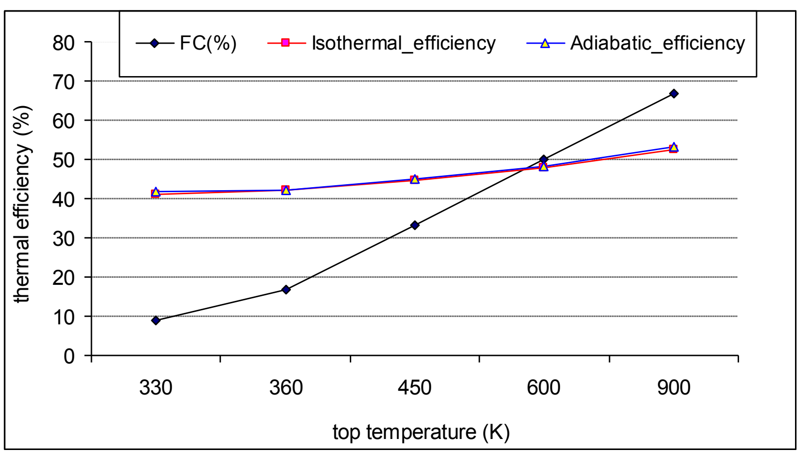



**Fig. 5. The thermal efficiency for the adiabatic and isothermal expansion based TC**

**Table 2. The thermal efficiency, CF and the specific work with helium as working fluid**

| CF (%) | $p_2$ (bar) | $T_2$ (K) | Isothermal TC | | Adiabatic TC | |
|---|---|---|---|---|---|---|
| | | | $W_I$ (kJ/kg) | $\eta_I$ (%) | $W_A$ (kJ/kg) | $\eta_A$ (%) |
| 9.10 | 1.1 | 330 | 65.33 | 41.13 | 39.16 | 41.92 |
| 16.70 | 1.2 | 360 | 136.33 | 42.16 | 78.90 | 42.21 |
| 33.30 | 1.5 | 450 | 379.00 | 44.78 | 210.00 | 44.90 |
| 50.00 | 2 | 600 | 863.80 | 48.02 | 452.00 | 48.30 |
| 66.70 | 3 | 900 | 2053.60 | 52.34 | 977.00 | 53.30 |

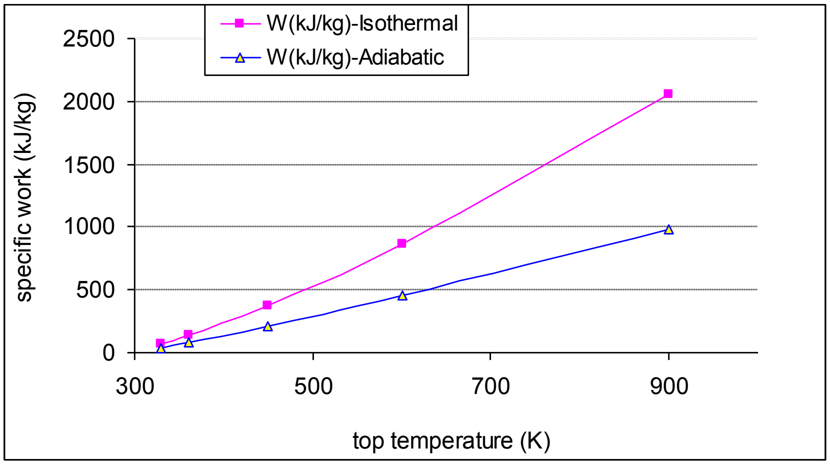



**Fig. 6. The specific work for the adiabatic and isothermal expansion based TC**

On the other hand, specific work is relevant in the decision making task: The specific work delivered by the TC operating with hydrogen as WF, is significantly greater than when operating with helium, which is de second in specific working capacity as depicted in Table 3. This characteristic may be useful to implement a thermal engine undergoing the minimum possible weight and size. Nevertheless, if size and weight are not relevant but thermal efficiency, then, helium should be selected as the best working fluid since:

- helium match the criterion of high thermal efficiency undergoing moderate high specific work, and
- hydrogen match the criterion of high specific work undergoing moderate high thermal efficiency.

Consequently, carbon dioxide and nitrogen should be discarded due to the lack of the relevant characteristics of the helium and the hydrogen.

**Table 3. Comparison of the thermal efficiencies and the specific works for different working fluids**

| $p_2$ (bar) | $T_2$ (K) | $W_I$ (kJ/kg) | $\eta_I$ (%) | $W_A$ (kJ/kg) | $\eta_A$ (%) | CF (%) |
|---|---|---|---|---|---|---|
| | | | **$CO_2$** | | | |
| 1.1 | 330 | 6.05 | 22.94 | 4.30 | 21.50 | 9.10 |
| 1.2 | 360 | 12.57 | 23.28 | 8.60 | 21.10 | 16.70 |
| 1.5 | 450 | 34.73 | 24.09 | 20.80 | 19.25 | 33.30 |
| 2 | 600 | 78.86 | 25.05 | 43.58 | 18.70 | 50.00 |
| | | | **$N_2$** | | | |
| 1.1 | 330 | 9.37 | 29.57 | 6.65 | 29.87 | 9.10 |
| 1.2 | 360 | 19.53 | 30.42 | 13.51 | 30.33 | 16.70 |
| 1.5 | 450 | 54.29 | 32.64 | 36.24 | 32.43 | 33.30 |
| 2 | 600 | 123.64 | 35.30 | 78.22 | 34.60 | 50.00 |
| | | | **He** | | | |
| 1.1 | 330 | 65.33 | 41.13 | 39.19 | 41.92 | 9.10 |
| 1.2 | 360 | 136.33 | 42.16 | 78.93 | 42.21 | 16.70 |
| 1.5 | 450 | 379.00 | 44.78 | 210.16 | 44.96 | 33.30 |
| 2 | 600 | 863.80 | 48.02 | 452.19 | 48.37 | 50.00 |
| | | | **$H_2$** | | | |
| 1.1 | 330 | 262.00 | 29.70 | 183.60 | 29.70 | 9.10 |
| 1.2 | 360 | 546.00 | 30.50 | 375.60 | 30.26 | 16.70 |
| 1.5 | 450 | 1517.00 | 32.70 | 1010.00 | 32.38 | 33.30 |
| 2 | 600 | 3458.00 | 35.60 | 2204.00 | 35.20 | 50.00 |

## 5. CONCLUSION

The study of closed systems based TCs undergoing an adiabatic expansion based transformation or an isothermal expansion based transformation to convert heat into mechanical work has been presented. As shown along the work, significant differences exist with respect to the quadrilateral Carnot based cycles.

The proposed TC has been studied for helium as working fluid considered as a real gas. Furthermore a comparison with other available WFs has been performed.

According to the characteristics of the proposed TC, new expressions for the thermal efficiency have been achieved and analysed.

The performance results of the TC have been compared with the results obtained for the Carnot cycle, both operating between the same range and ratio of temperatures.

Some conclusions are related to:

- the TC structure which is configured under three closed system based transformations,
- the thermal efficiency of the cycle which exceeds the Carnot efficiency under appropriated operating conditions for adiabatic and isothermal expansion based heat to work conversion.
- Due to the independence of the thermal efficiency with the CF, it is not a function of the top temperature as in the CF, so that the TC can efficiently operate at a relative low temperature even rendering high thermal efficiency.

Thus, the TC using helium as WF renders the highest thermal efficiency with regard to other studied WFs. For instance, the TC undergoing an isothermal heat to work conversion path function, for a top temperature $T_2$ of 330 K the thermal efficiency approaches 41.1% while CF is 9.1%, and for a $T_2$ of 450 K the thermal efficiency approaches 44.8% while CF is 33.0%. Furthermore, the corresponding specific works at supposed top temperatures is respectively 65.3 and 379 kJ/kg, since the specific work is a linear function of the top temperature.

On the other hand, according to the results, an alternative WF to the helium is the hydrogen, since the TC undergoing an isothermal heat to work conversion path function, for a top temperature $T_2$ of 330 K the thermal efficiency approaches 29.7% while CF is 9.1%, and for a $T_2$ of 450 K the thermal efficiency approaches 32.7% while CF is 33.0%. However, the corresponding specific works at supposed top temperatures is respectively 262.0 and 1517.0 kJ/kg, which means a very high difference with respect to the helium, which contributes to the dramatic reduction of the size and weight of the thermal engine. As shown in Table 3, the specific work is significantly lower for the TC undergoing an adiabatic heat to work conversion path function.

Furthermore, thermal losses due to isentropic efficiency are avoided if reversible transformations can be approached.

As shown by the results, the ideal thermal efficiency is significantly increased in comparison with the actual conventional ORCs. An overall conclusion concerns to the fact that a new family of efficient power plants operating at low temperatures using renewable heat resources as well as solar low temperature thermal, geothermal or residual heat is feasible.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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## APPENDIX A. THE DATA FOR THE CASE STUDY

**Table A1. Data applied on the case study operating with He as working fluid using data from [20]**

| Point | T (K) | u (kJ/kg) | s (kJ/kg.K) | p (bar) | v ($m^3$/kg) |
|---|---|---|---|---|---|
| | | **Adiabatic expansion based TC** | | | |
| 1 | 300.00 | 940.00 | 28.030 | 1.0 | 6.2345 |
| 2 | 330.00 | 1033.50 | 28.320 | 1.1 | 6.2345 |
| 3 | 317.44 | 994.34 | 28.320 | 1.0 | 6.5967 |
| 2 | 360.00 | 1127.00 | 28.595 | 1.2 | 6.2345 |
| 3 | 334.70 | 1048.10 | 28.595 | 1.0 | 6.9553 |
| 2 | 450.00 | 1407.40 | 29.290 | 1.5 | 6.2345 |
| 3 | 383.60 | 1197.50 | 29.290 | 1.0 | 7.9509 |
| 2 | 600.00 | 1874.90 | 30.190 | 2.0 | 6.2345 |
| 3 | 455.00 | 1423.10 | 30.190 | 1.0 | 9.4549 |
| 2 | 900.00 | 2810.00 | 31.450 | 3.0 | 6.2345 |
| 3 | 580.00 | 1812.40 | 31.450 | 1.0 | 12.0500 |
| | | **Isothermal expansion based TC** | | | |
| 1 | 300 | 940.00 | 28.027 | 1 | 6.23450 |
| 2 | 330 | 1033.50 | 28.324 | 1.1 | 6.23450 |
| 3 | 330 | 1033.50 | 28.973 | 1 | |
| 2 | 360 | 1127.00 | 28.60 | 1.2 | 6.23450 |
| 3 | 360 | 1127.00 | | 1 | |
| 2 | 450 | 1407.40 | 29.29 | 1.5 | 6.23450 |
| 3 | 450 | 1127.00 | | 1 | |
| 2 | 600 | 1874.90 | 30.19 | 2 | 6.23450 |
| 3 | 600 | 1127.00 | | 1 | |
| 2 | 900 | 2810.00 | 31.45 | 3 | 6.23450 |
| 3 | 900 | 2809.00 | 31.45 | 1 | |



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