British Journal of Applied Science & Technology
16(2): 1-5, 2016, Article no.BJAST.26148
ISSN: 2231-0843, NLM ID: 101664541

SCIENCEDOMAIN *international*
www.sciencedomain.org

Upper Critical Field of Superconducting Magnesium Diboride (MgB_2)

Habte Dulla Berry[1*]

[1]Department of Physics, Dilla University, College of Natural and Computational Sciences, P.O.Box 419, Dilla, Ethiopia.

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/BJAST/2016/26148
Editor(s):
(1) Joao Miguel Dias, Habilitation in Department of Physics, CESAM, University of Aveiro, Portugal.
Reviewers:
(1) A. Ayeshamariam, Khadir Mohideen College, Adirampattinam, India.
(2) Subramaniam Jahanadan, Kolej Matrikulasi Labuan (KML), Malaysia.
Complete Peer review History: http://sciencedomain.org/review-history/14730

Original Research Article

Received 2nd April 2016
Accepted 7th May 2016
Published 22nd May 2016

ABSTRACT

In this paper the author addresses the nature of upper critical field of magnesium dioxide (MgB_2) using Ginzberg- Landau (GL) approach. Nowadays, Superconducting materials show promising progress for the technological applications. According to this and previous studies, magnesium diboride is one of the superconducting materials which has relatively better critical temperature. The work will focus on determination of critical field and temp. of the material.

Keywords: Critical field; critical temperature; Ginzburg-Landau free energy; superconductor.

1. INTRODUCTION

Superconductivity was first discovered in 1911 by the Dutch physicist, Heike Kammerlingh Onnes and he was dedicated his scientific career to exploring `extremely cold refrigeration [1]. Onnes passed a current through a very pure mercury wire and measured its resistance as he steadily lowered the temperature. Much to his surprise there was leveling off of resistance, let alone the stopping of electrons as suggested by Kelvin. At 4.2 k the resistance suddenly vanished. Current was flowing through the mercury wire and nothing was stopping it. After the fascinating discovery of high temperature superconductivity at 35k in La-Ba-Cu-O by Bednorz and Muller [2]

**Corresponding author: E-mail: habtedul@yahoo.com, habtix07@gmail.com;*

in 1986, superconductivity was discovered in Y-Ba-Cu-O system at 90k by Wu et al. [3]. With the discovery of two new superconductors Bi-Sr-Ca-Cu-O(T_c 110 k) [4] and Ti-Ba-Ca-Cu-O(Tc 125 k) [5] a new dimension has been added to the field of ever increasing T_c materials. Although many of their properties have been studied extensively during the last few years, the mechanism of superconductivity remains illusive.

2. MODEL

In this work we use the generalized Ginzburg-Landau equations to solve for the upper critical field of magnesium diboride and phase transition temperature. The well known phenomenological theory of superconductivity, proposed by Ginzburg and Landau [6-7] much before the microscopic theory of Bardeen, Cooper and Schriefer [8] has been quite successful in describing the behavior of conventional superconductors. It is based on the general theory of second-order phase transition [9] assuming the existence of an order parameter which is non zero in the ordered (superconducting) state and zero in the disordered (normal) state.

Maining three terms describing the superconducting effects. In the absence of fields and gradients, we have $F_s - F_n = \alpha|\psi\ |^2 + \frac{\beta}{2}|\psi\ |^4$ which can be viewed as a series expansion in powers of or, in which only the first two terms are retained.

2.1 Ginzburg-Landau free Energy and its Relation to Upper Critical Field

In the presence of two order parameters ψ_1 and ψ_2, in a superconductor, the Ginzburg-Landau free energy can be written as [10].

$$F(\psi_1,\psi_2) = \int d^3r\,(F(\psi_1) + F(\psi_2) + F_{12}(\psi_1,\psi_2) + \frac{H^2}{8\pi}). \qquad (2.1)$$

With $F_j(\psi_j) = \frac{\hbar^2}{4m_j}|(\nabla - \frac{2\pi iA}{\phi_0})\psi_j|^2 + \alpha_j(T)\psi_j^2 + \frac{\beta}{2}\psi_j^4$, which means

$$F_1(\psi_1) = \frac{\hbar^2}{4m_1}|(\nabla - \frac{2\pi iA}{\phi_0})\psi_1|^2 + \alpha_j(T)\psi_1^2 + \frac{\beta}{2}\psi_1^4$$

and

$$F_2(\psi_2)\alpha_j = \frac{\hbar^2}{4m_2}|(\nabla - \frac{2\pi iA}{\phi_0})\psi_2|^2 + \alpha_j(T)\psi_2^2 + \frac{\beta}{2}\psi_{2,}^4$$

these are free energies of the two separate bands of superconducting MgB2.

$$F_{12}(\psi_1,\psi_2) = \varepsilon(\psi_1\psi_2^* + cc) + \varepsilon_1[\left(\nabla + \frac{2\pi iA}{\phi_0}\right)\psi_1^*\left(\nabla - \frac{2\pi iA}{\phi_0}\right)\psi_2 + cc] \qquad (2.2)$$

Where $F_{12}(\psi_1,\psi_2)$ is free energy of the mixed state and $\varepsilon, \varepsilon_1$ are the interband mixing of two order parameters and their gradients respectively. $\vec{H}$ is the external magnetic field $\vec{H} = \nabla\text{xA}$, α_j, β_j, m_j are the temperature dependent coefficient, temperature - in dependent coefficient and mass of the carriers in band j(j = 1, 2) respectively. Let's assume that there is only magnetic field in the z-direction, $\vec{H} = H\hat{k}$ and vector potential is in y-direction, $\vec{A} = Hx\hat{\jmath}$ that $H = |\vec{H}|$ and x is displacement. Equilibrium or the minimum value of the free energy given in eq. (2.2) with respect to the order parameters ψ_1^* and ψ_2^* respectively we get [11-20].

$$\frac{\partial F}{\partial\psi_1^*} = \frac{\hbar^2}{4m_1}\left(\nabla + \frac{2\pi iA}{\phi_0}\right)\psi_1\left(\nabla - \frac{2\pi iA}{\phi_0}\right) + \alpha_1(T)\psi_1 + \varepsilon\psi_2 + \varepsilon_1\left(\nabla + \frac{2\pi iA}{\phi_0}\right)\left(\nabla - \frac{2\pi iA}{\phi_0}\right)\psi_2 = 0, \qquad (2.3)$$

$$\frac{\partial F}{\partial\psi_2^*} = \frac{\hbar^2}{4m_2}\left(\nabla + \frac{2\pi iA}{\phi_0}\right)\psi_2\left(\nabla - \frac{2\pi iA}{\phi_0}\right) + \alpha_2(T)\psi_2 + \varepsilon\psi_1 + \varepsilon_1\left(\nabla + \frac{2\pi iA}{\phi_0}\right)\left(\nabla - \frac{2\pi iA}{\phi_0}\right)\psi_1 = 0 \qquad (2.4)$$

where ϕ_0 is the quantum flux. Mathematically it can be represented by $\phi_0 = \frac{2\pi\hbar c}{e}$. For one dimensional case the above two equations that are equation (2.4) and (2.5) can be reduced to

$$\frac{\hbar^2}{4m_1}(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2})\,\psi_1 + \alpha_1(T)\psi_1 + \varepsilon\psi_2 + \varepsilon_1(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2})\,\psi_2 = 0 \qquad (2.5)$$

$$\frac{\hbar^2}{4m_2}(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2})\,\psi_2 + \alpha_2(T)\psi_1 + \varepsilon\psi_2 + \varepsilon_1(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2})\,\psi_1 = 0 \qquad (2.6)$$

Where $l_s = \frac{\hbar c}{2He}$ and the field is H_{c2}. Now, after some simple calculations we arrive to:-

$$H_{c2} = \frac{\frac{\pi}{\phi_0}\left(\frac{1}{2\varepsilon_1^2} - + \frac{1}{2\varepsilon_2^2} + \frac{k}{\varepsilon_{12}^2}\right) \pm (\left(\frac{\pi}{\phi_0}\left(\frac{1}{2\varepsilon_1^2} - + \frac{1}{2\varepsilon_2^2} + \frac{k}{\varepsilon_{12}^2}\right)^2\right) - 4(1-k^2)\frac{1\pi^2}{4\phi_0^2}\left(\frac{1}{\varepsilon_1^2\varepsilon_2^2} - \frac{1}{\varepsilon_{12}^2}\right))^{1/2}}{\frac{2\pi^2\,(1-k^2)}{\phi_0^2}} \qquad (2.7)$$

Which is equal to

$$H_{c2}=\frac{\phi_0}{2\pi(1-k^2)}\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)\pm\frac{\phi_0}{2\pi(1-k^2)}(\left(\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)^2\right)-(1-k^2)\left(\frac{1}{\varepsilon_1^2\varepsilon_2^2}-\frac{1}{\varepsilon_{12}^2}\right))^{1/2} \quad (2.8)$$

Equation (2.8) has two terms in square root that we can make the approximation by considering these two terms. It can be considered in to cases.

Case I when $\left(\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)^2\right)\gg(1-k^2)\left(\frac{1}{\varepsilon_1^2\varepsilon_2^2}-\frac{1}{\varepsilon_{12}^2}\right)$.

Now let's apply important application of the Taylor and Maclaurin expansions is the derivation of the binomial theorem for negative and/or non integral powers .i. e is let $f(x)=(1+x)^m$, in which **m** may be negative and is not limited to integral values. Then,

$$(1+x)^m=1+mx+\frac{m(m-1)}{2!}x^2+\ldots \quad (2.9)$$

By using the approximation in case I, we can write equation (**2.9**) as follows

$$H_{c2}=\frac{\phi_0}{2\pi(1-k^2)}\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)\pm\frac{\phi_0}{2\pi(1-k^2)}(\left(\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)^2\right)(1-\frac{(1-k^2)\left(\frac{1}{\varepsilon_1^2\varepsilon_2^2}-\frac{1}{\varepsilon_{12}^2}\right)}{\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}})^{1/2} \quad (2.10)$$

By substituting eq.(2.8) in eq.(2.10):-

$$H_{c2}=\frac{\phi_0}{2\pi(1-k^2)}\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)\pm\frac{\phi_0}{2\pi(1-k^2)}(\left(\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)^2\right)(1-(1-k^2)\left(\frac{1}{\varepsilon_1^2\varepsilon_2^2}-\frac{1}{\varepsilon_{12}^2}\right)\frac{1}{\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)^2}+\cdots$$

Therefore we have two values of H_{c2} that are $H_{c2}(-)$ and $H_{c2}(+)$, i.e.

$$H_{c2}(-)=\frac{\phi_0}{\pi(1-k^2)}\left(\frac{1}{2\varepsilon_1^2}\mp\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)+\frac{1}{2}\frac{\phi_0}{2\pi}(\left(\left(\frac{1}{\varepsilon_1^2\varepsilon-2^2}-\frac{1}{\varepsilon_{12}^4}\right)^2\right)\frac{1}{\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)^2}+\cdots \quad (2.11)$$

and $H_{c2}(+)$ will have

$$H_{c2}(+)=\frac{\frac{1}{2}\frac{\phi_0}{2\pi}\left(\frac{1}{\varepsilon_1^2\varepsilon_2^2}-\frac{1}{\varepsilon_{12}^4}\right)}{\left(\frac{1}{2\varepsilon_1^2}-+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)} \quad (2.12)$$

Case II for $\left(\frac{1}{2\varepsilon_1^2}+\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)^2\ll(1-k^2)\left(\frac{1}{\varepsilon_1^2\varepsilon_2^2}-\right.$ *1ε122,* when we use this case the term in the square root becomes complex. Therefore we can conclude that the second case is in valid because we don't have complex field rather it is real

$$H_{c2}(-)=\frac{\phi_0}{2\pi(1-k^2)}\left(\frac{1}{2\varepsilon_1^2}\mp\frac{1}{2\varepsilon_2^2}+\frac{k}{\varepsilon_{12}^2}\right)-\frac{\phi_0}{2\pi}\left(\frac{1}{\varepsilon_{12}^4}-\frac{1}{\varepsilon_1^2\varepsilon_2^2}\frac{1}{(1-k^2)}\right))^{1/2}+\ldots \quad (2.13)$$

Now, from case I and case II we have obtained two equations for the upper critical field (H_{c2}) that are equations (2.12) and (2.13). Next step, we will check the conditions to get physical solution of (H_{c2}). We have the critical field of one band model as $H_{c2}=\frac{\phi_0}{2\pi\varepsilon^2}$. Just to obtain $H_{c2}=\frac{\phi_0}{2\pi\varepsilon^2}$ from two-band model we have used $\alpha_2=\varepsilon=\varepsilon_1=0$ so, from the above mentioned equations only equation (2.12) is reduced to $H_{c2}=\frac{\phi_0}{2\pi\varepsilon^2}$ that means this equation can be the solution of two band model. Finally, the analytical equation of upper critical field can be written as:

$$H_{c2}=\frac{\phi_0}{2\pi\varepsilon_{12}^2}\left(\frac{\varepsilon_{12}^2}{\varepsilon_1^2}+\frac{\varepsilon_{12}^2}{\varepsilon_2^2}+2k\right)\left(\frac{1}{1-k^2}\right)-\frac{(\frac{\varepsilon_{12}^4}{\varepsilon_1^2\varepsilon_2^2}-1)}{(\frac{\varepsilon_{12}^2}{\varepsilon_1^2}+\frac{\varepsilon_{12}^2}{\varepsilon_2^2}+2k)^2} \quad (2.14)$$

The effect of anisotropy mass tensor on the upper critical field is included by replacing ε in equation (2.14) with coherence length tensor { ε}. The calculation procedure is as same as above. Then we get

$$H_{c2} = \frac{\phi_0}{2\pi\varepsilon_{12}^2(sin^2\theta+\delta^2cos^2\theta)}\left(\frac{\varepsilon_{12}^2}{\varepsilon_1^2}+\frac{\varepsilon_{12}^2}{\varepsilon_2^2}+2k\right)\left(\frac{1}{1-k^2} - \frac{\left(\frac{\varepsilon_{12}^4}{\varepsilon_1^2\varepsilon_2^2}-1\right)}{\left(\frac{\varepsilon_{12}^2}{\varepsilon_1^2}+\frac{\varepsilon_{12}^2}{\varepsilon_2^2}+2k\right)^2}\right) \quad (2.15)$$

Where θ is the angle between magnetic field and the layer of superconductor. $\delta = (\frac{m}{M})^{1/2}$

3. RESULTS AND DISCUSSION

We have used the Ginzburg-Landau theory in order to obtain expression for upper critical field of two band superconducting MgB2. Therefore, in this case we do have two order parameters, ψ_1 and ψ_2 , based on this the field can be written as follows.

$$H_{c2} = \frac{\phi_0}{2\pi\varepsilon_{12}^2}\left(\frac{\varepsilon_{12}^2}{\varepsilon_1^2}+\frac{\varepsilon_{12}^2}{\varepsilon_2^2}+2k\right)\left(\frac{1}{1-k^2}\right) - \frac{\left(\frac{\varepsilon_{12}^2}{\varepsilon_1^2\varepsilon_2^2}-1\right)}{\left(\frac{\varepsilon_{12}^2}{\varepsilon_1^2}+\frac{\varepsilon_{12}^2}{\varepsilon_2^2}+2k\right)^2} \quad (3.1)$$

where the parameters have their usual meaning such as the coherence length (ε) quantum flux (ϕ_0) and others. Which reduces to $H_{c2} = \frac{\phi_0}{2\pi\varepsilon^2}$ for single band case as expected by using $\alpha_2 = \varepsilon = \varepsilon_1 = 0$. Since MgB_2 is considered as a two-band s-wave superconductor, it shows the layered property. In the layered superconductors, since the overlap of electron wave function is larger within the layers than between layers, it can be assumed that the electrons have a high effective mass formation normal to the layers and a low effective mass formation with in a layer. If we know the values of anisotropy parameters, we can get the value of Ω to simplify, we will consider as Ω a constant parameter. The assumption that the upper critical field in ab-plane (H_{C2}^{ab}) and c-axis (H_{C2}^{c}) can be found by set of the suitable parameters. We write the magnetic field in the form that $H_{c2} = H_{c2}(a_1, a_2, \Omega, \mathrm{k})$ where $a_1 = \frac{\hbar^2}{2m\lambda_1}$, $a_2 = \frac{\hbar^2}{2m\lambda_2}$. After fitting the experimental data of upper critical field of single crystal MgB2, the upper critical field in ab-plane (H_{C2}^{ab}) is $H_{C2}^{ab} = H_{c2}(6.5,6.5,0.7,0.5)$ and in c-axis (H_{C2}^{c}) is $H_{C2}^{c} = H_{c2}(13,13,0.7,0.5)$ as shown in Fig. 1. In Fig. 2 the ratio of upper critical field ($\gamma = \frac{H_{C2}^{ab}}{H_{C2}^{c}} = \frac{H_{c2}(6.5,6.5,0.7,0.5)}{H_{c2}(13,13,0.7,0.5)}$) versus temperature is shown. The maximum γ at $T = 0k$ is equal to 2.5.

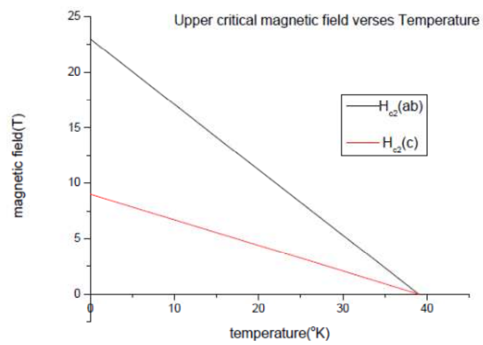

Fig. 1. Upper critical magnetic field versus temperature

As we can see from this figure, the critical magnetic field decays with increasing temperature i.e. in consistent with previous studies.

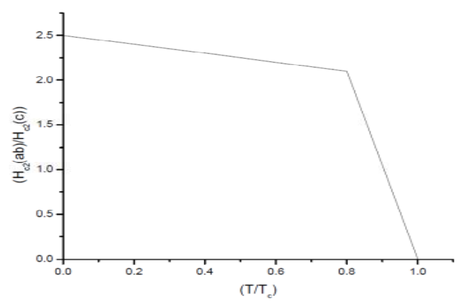

Fig. 2. Anisotropic property of magnetic field versus temperature

In this case also, the figure compares the relationship between anisotropic critical field and temperature. Therefore, the result is as we expected in line with prior studies.

4. CONCLUSION

In this study, we used two- band Ginzburg-Landau approach in order to determine the upper critical magnetic field. According to the mathematical calculation, the magnetic field depends on anisotropy of order parameter. This indicates that the critical field along ab-plane (H_{c2}^{ab}) is quite different from c-axis H_{c2}^{c}. In general, the upper critical field relay on temperature and decreases with when the temperature goes to higher and higher.

Therefore, the coherence length is strongly temperature dependent, so that it can cause temperature dependency and anisotropy nature on the upper critical field of MgB_2. In addition to this in this study we tried to plot upper critical field versus temperature and anisotropy critical field versus temperature. The results of this theoretical study more or less fits with experimental findings.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Efthimias Kaxiras. Atomic and electronic structure of solids, (Cambridge University Press, Cambridge); 2003.
2. Bednor JG, Muller KA. Possible high Tc superconductivity in the Ba La Cu 0 System. Z. Phys. 1986;B.64:184.
3. Meng RL, Geo L, Huang ZJ, Wang YQ, Chu CW. Superconductivity at 93 K in a new mixed-phase Y-Ba-Cu-O compound system at ambient pressure. Phys. Rev. Lett. 1987;58:908.
4. Maeda H, Tanaka Y, Fukutomi M, Asano TA. A new high Te oxide superconductor without rear earth element. Jpn. J. Appl Phys. 1988;27:L209.
5. Sheng ZZ, Hermann NH, El Ali A, Almason C, Estrada J, Datta T, Matson RJ. Superconductivity at 90 K in the Tl-Ba-Cu-O system. Phys. Rev. Lett. 1988;60,937.
6. Cyrot M. Ginzburg-Landau theory for superconductors Rep. Prog. Phys. 1973; 36:103.
7. Ginzburg VL, Landau LD. On the theory of superconductivity, Zh. Exsp. Theor. Fiz. 1950;20:1964.
8. Bardeen J, Cooper L, Schrieffer JK. Theory of Superconductivity. Phys. Rev. 1957;108:162.
9. Landau LD, Lifshitsz EM. Statistical physics, 2nd Ed. (Pergamon, Oxford; 1969).
10. Singh P, Sinha KP. A possible mechanism of high T_c superconductivity involving biexcitons, solide state commun. 1990;73,45.
11. Askerzada IN. Ginzburg–Landau theory for two-band s-wave superconductors: Application to non-magnetic borocarbides $LuNi_2B_2C$, YNi_2B_2C and magnesium diboride MgB_2. Phsica C. 2003;397:99.
12. Askerzade IN, Gencer A, Guclu N, Kiliç A. Two-band Ginzburg–Landau theory for the lower critical field H_{c1} in MgB_2 supercond. Sci. Technol. 2002;15:L13-L16.
13. Posazhennikova AI, Dahm T, Maki K, Anisotropic *s*-wave superconductivity: Comparison with experiments on MgB_2 single crystals. Euro Phys. Lett. 2002;60: 134.
14. Dahm T, Schopohl N. Fermi surface topology and the upper critical field in two-band superconductors: Application to MgB2. Phys. Rev. Lett. 2003;91:017001.
15. Miranovic P, Machida K, Kogan VG. Quasi-classical calculation of the mixed-state thermal conductivity in s- and d-wave superconductors. J. Phys. Soc. Jpn. 2003;72:221.
16. Koshelev AE, Golubov AA. Why magnesium diboride is not described by anisotropic Ginzburg-Landau theory. Physical Review Letter. 2004;92:107008.
17. Dao VH, Zhitomirsky ME. Anisotropy of the upper critical field in MgB_2: The two-gap Ginzburg-Landau theory. European Physical Journal. 2005;B 44:183.
18. Jourdan M, Huth M, Adrian H. Superconductivity mediated by spin fluctuations in the heavy-Fermion compound UPd2Al3, Nature (London). 1999;398:47.
19. Haas S, Maki K. Anisotropic *s*-wave superconductivity in MgB2. Phys. Rev. B. 2002;65:020502(R).
20. Nagamatsu J, et al. Superconductivity at 39K in magnesium diboride. Nature. 2001;410:63.

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/14730